

The effectiveness of cramer's rule in improving mathematical thinking skills and learning outcomes in solving systems linear equation of fixed pulley

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Abstract

Learning innovation is needed to make the learning process more attractive and improve student learning outcomes. Learning outcomes can be improved by making learning more interesting and developed according to student needs. Learning materials are packaged with different problem-solving methods that make learning more interesting to improve student learning outcomes. Based on existing findings, students' mathematical thinking skills are still relatively low. This study aims to measure the effectiveness of applying Cramer's rule to improve mathematical thinking skills and learning outcomes. This study uses a quantitative descriptive method which was carried out at Jember State High School and the Physics Education Study Program, Jember University in the odd semester of the 2024/2025 academic year. The results of data analysis from mathematical thinking skills affect the success of physics learning. The results of the homogeneity test and normality test that have been carried out show that the samples used are homogeneous and normal. The N-Gain value is obtained in the high category for improving mathematical thinking skills and learning outcomes. And the student response is very positive towards the use of Cramer's rule in solving the system of linear equations on the fixed pulley material. So, it can be concluded that Cramer's rule in solving linear equation systems on pulley material remains effective in improving mathematical thinking skills and learning outcomes.

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1. Introduction

21st century education demands the development of students' potential in learning so that learning innovation is needed in learning to provide changes in teaching and learning activities. Learning innovation makes learning attractive to make students comfortable and get maximum learning outcomes (Prastika., 2021; Rahayu et al, 2022). One important factor in improving learning outcomes is interest in learning, low interest in learning results in low learning outcomes (Ndraha & Mendrofa, 2022). Interest in learning is influenced by several factors, including interest, motivation, satisfaction, and needs (Chen et al., 2021). Interest in learning can be increased by making learning more interesting and developed according to student needs (Hamidah & Setiawan, 2019); (Sulistiyorini & Listiadi, 2022). Learning materials that are packaged with different concepts or solving problems with different methods (alternatives) are one way to make learning more interesting besides choosing the right learning model and learning media (Taufik 2024).

Physics learning develops analytical, inductive and deductive thinking skills in solving problems qualitatively and quantitatively using mathematics, and develops knowledge, skills and self-confidence (Siombone & Niwele, 2023). Physics problems such as presenting experimental results, formulas, calculations, graphs and conceptual explanations of a natural phenomenon are expressed in the form of simple mathematical equations to complex mathematical equations (Ornek et al., 2008). So that solving physics problems uses many mathematical concepts, even mathematics is seen as the language of physics that has a function to formulate problems in mathematical models (Neumann et al., 2021). Mathematical ability is not just the ability to count, but students must master mathematical thinking skills (Anwar., 2018). Improving students' abilities including critical

mathematical thinking skills in physics learning is very important. Mathematical thinking skills in learning can improve understanding and increase the appeal of learning (Drijvers et al., 2019).

The mathematical thinking ability and mathematical literacy ability of students in Indonesia are still low compared to several neighboring countries. Based on PISA 2019 data, students' ability to model problems mathematically, choose, compare and evaluate problem-solving strategies is only 1%, far below the global average of 11% (OECD, 2019). The results of the PISA survey in 2022 recorded that the mathematical literacy ability of students in Indonesia was ranked 70th out of 81 countries surveyed (Yudhoyono et al., 2024). The results of a study by Tupulu et al (2023) stated that students' mathematical thinking ability is still quite low, as evidenced by data from students who did not complete the problem solving by 59%. Meanwhile, Sari et al (2021) stated that the mathematical thinking ability in the specializing indicator was quite good, but for the other three indicators (generalizing, conjecturing, and convincing) it was still low. Basic mathematical skills (Saraswati et al., 2019) and mathematical thinking skills (Woitkowski., 2020) greatly influence the success of physics learning. Low mathematical thinking skills have an impact on low student learning outcomes, especially in physics subjects.

One of the physics materials that has low learning outcomes because it is considered difficult and requires high mathematical skills is the application of Newton's 2nd law (Rizkita & Mufit., 2022). The percentage of students in mastering Newton's Law material is 26.7 in the low category. The causes of low mastery of Newton's 2nd Law material are due to several aspects, one of which is low mathematical mastery, lack of practice in working on questions and low learning motivation (Putra & Heriyanto., 2020). The application of Newton's 2nd law to the free force diagram of a fixed pulley system object produces a Linear Equation System of at least two variables. Solving the Linear Equation System of at least two variables and Linear Equation System of three variables in physics problems generally uses the substitution and elimination methods. This method is quite effective in solving the Linear Equation System of at least two variables and is less effective in solving LES with more than 2 variables because it is not time efficient and has a high error rate (Ilmi et al., 2023). The process of solving with the substitution and elimination methods that must be done repeatedly can take a relatively long time and is prone to errors in manual calculations. These problems can be overcome by using a more efficient alternative method, namely the matrix method in solving linear equation system problems.

From the free force diagram in the fixed pulley system by considering the rotational motion of the pulley, 3 linear equations will be obtained, as shown in equation (1).

$$\begin{aligned} a_1x_1 + b_1x_2 + c_1x_3 &= h_1 \\ a_2x_1 + b_2x_2 + c_2x_3 &= h_2 \\ a_3x_1 + b_3x_2 + c_3x_3 &= h_3 \end{aligned} \quad (1)$$

Linear Equation System can be expressed in the form of matrix multiplication, where the variable coefficients and constants of each equation are arranged in matrix form (Ayu et al., 2024). So that equation (1) can be expressed in the matrix equation matrix as shown in equation (2).

$$AX = H \quad (2)$$

where ; and $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$; $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ $H = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}$ is coefficient matrix, variable matrix and matrixconstant (Pratiwi., 2024). The use of matrix multiplication in Linear Equation System (LES) statements is presented in a more concise and systematic form, making it easier to analyze mathematical calculations (Busrah., 2019). Solving LES in matrix multiplication can be done analytically and numerically through the Gauss Elimination Method, Cramer's Rule, Matrix Inverse Method, and Gauss-Jordan Elimination Method (Arnas & Anam., 2019). The substitution method, Cramer's rule, and Gauss elimination method have advantages and disadvantages in solving LES (Luo et al., 2021).

Cramer's rule is one of the methods used in solving LES. This method can facilitate mathematical solutions (Ilmi et al., 2023). Cramer's rule is the most popular method (Bramasto & Khairiani, 2022) and is also effective (Devita, 2022) in solving LES. Cramer's rule is an alternative method in solving LES in physics problems (Aminah & Radita., 2020). The application of Cramer's rule can solve 2-loop electrical circuits (Krisnawanto et al, 2024); (Wahyudianti et al., 2023). The application of Cramer's rule in solving LES has been shown to increase learning motivation (Simamora, 2022) and can also improve learning outcomes (Aida et al., 2023). Cramer's rule also received a very positive response in solving 2-loop electrical circuit problems (Krisnawanto 2024). The following are the stages of using Cramer's rule in LES:

(1) Changing the LES in-matrix multiplication as equation (2).

(2) Determine the determinant of matrices A, B, C and D, with

$$A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}; B = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}; C = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}; \text{ dan } D = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

(3) Determine the values of variables x, y, and z using Cramer's rule (Alhasan, 2021), namely as shown in equation (3).

$$x_1 = \frac{\det B}{\det A}, \quad x_2 = \frac{\det C}{\det A}, \quad \text{ dan } \quad x_3 = \frac{\det D}{\det A} \quad (3)$$

The application of Cramer's rule has been proven to be very effective in solving LES and has also received a very positive response in solving 2-loop electrical circuit problems. The application of Newton's II law to the free force diagram on an object in a pulley system will produce an LES of two or more variables. The effectiveness of an action or method in the learning process is measured by the achievement of the expected learning outcomes (Norman et al., 2022). Based on this description, this study aims to describe how to improve mathematical thinking skills and learning outcomes and student responses after applying Cramer's rule in solving problems on the fixed pulley system material? The novelty of this study is explaining the steps for applying Cramer's rule in solving LES on the fixed pulley system material which can later be used as an alternative method in solving problems in the learning process besides using the substitution and elimination methods.

2. Method

The research was conducted at one of Jember's Senior High Schools and the Physics Education Study Program of Jember University using Pre-Experimental Design with the form of One Group Pretest-Posttest as shown in Table 1.

Table 1. One Group, Pretest and Posttest Design

Group	PreTest	Treatment	PostTest
Experiment	O ₁	X	O ₂

Information:

O₁= Giving initial tests to the experimental class

O₂= Giving final test to experimental class

X = Learning by implementing Cramer's rule on the Fixed Pulley Linear Equation System (LES) (Source:Manly, 1992).

Sampling using purposive sampling technique using experimental class. The sample used was 30 students and 30 students. Data on mathematical thinking ability and learning outcomes were obtained through pretest and posttest using 5 essay questions of fixed pulley system. The instruments in this study included pretest and posttest questions and a rubric for assessing mathematical thinking ability and a response questionnaire that had been validated and declared valid by 3 validators (2 lecturers and 1 physics teacher) to measure the improvement of mathematical thinking ability and learning outcomes and student and student responses. This instrument has also been tested in other classes in one of the same courses and gave the same results. The indicators of mathematical thinking ability in this study are specializing, generalizing, conjecturing, and convincing as shown in Table 2.

Table 2. Mathematical Thinking Ability Indicators

Indicator	Description
Specializing	Write the known, asked, and unit parts completely and correctly.
Generalizing	Describe the direction of force along with information and write linear equations correctly and completely.
Conjecture	Substituting linear equations into matrices, calculating the acceleration and tension values of the rope using Cramer's rule assisted by Sarrus correctly and completely.
Convincing	Determine the answer, units, and contain information correctly and completely

(Ferdianto et al. 2022; Van Es & Sherin, 2002).

The data analysis technique in this study used the N-Gain Score test through the SPSS 25 and MS Excel applications with the equation (4).

$$N_{gain} \langle g \rangle = \frac{N_f - N_i}{N_{max} - N_i} \quad (4)$$

Information :

N_f = Posttest mean score

N_i = Average pretest score

N_{max} = Maximum value

N_{gain} test results are analyzed using the percentage criteria shown in Table 2.

Table 3. Level Categories $N_{gain} \langle g \rangle$

Limitation	Category
$\langle g \rangle \geq 0,7$	Hight
$0,3 < \langle g \rangle < 0,7$	currently
$\langle g \rangle < 0,3$	low

Modified from Hake (1999)

Student response data was obtained through a response questionnaire using 5 indicators of interest, motivation, interest, satisfaction, and response (Nurlatipah et al., 2015) with 20 statements. Response analysis using a Likert scale and calculated using the equation (5).

$$Percentage = \frac{\text{total score obtained}}{\text{highest score}} \times 100\% \quad (5)$$

The percentage results (P) can be classified based on the response score criteria in table 4.

Table 4. Percentage of responses

Response percentage interval	Criteria
$80\% \leq P < 100\%$	Very positive
$60\% \leq P < 80\%$	Positive
$40\% \leq P < 60\%$	Quite positive
$20\% \leq P < 40\%$	Less positive
$P < 20\%$	Very less positive

Arikunto (2020)

3. Results and Discussion

3.1. Cramer's Rule on Fixed Pulley LES

Solving problems with fixed pulley systems, for example two objects each has its own mass $m_1 = 3 \text{ kg}$ and which is connected by a very light and inelastic rope through a fixed pulley. Assume the acceleration due to gravity and the pulley is a hollow cylinder with mass and radius. To determine the acceleration of the object's motion and the tension in the rope, start with $m_2 = 5 \text{ kg}$; $g = 10 \text{ m/s}^2$; $M = 2 \text{ kg}$; $r = 10 \text{ cm}$ by depicting the free style diagram as illustrated in Figure 1.

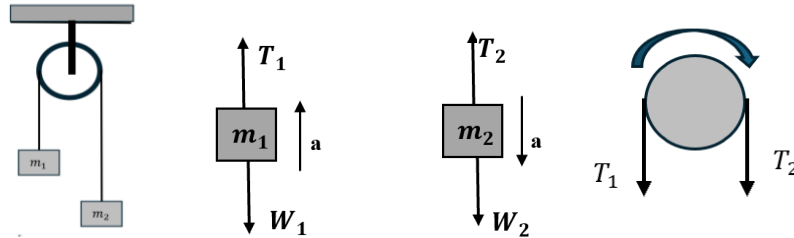


Figure 1. Fixed Pulley System and Free Force Diagram

By applying Newton's 2nd law to each object in the free force diagram, the LES is obtained as follows:

$$\text{Object 1 : } T_1 - 3a = 30$$

$$\text{Object 2 : } T_2 + 5a = 50$$

$$\text{Pulley : } T_2 - T_1 - 2a = 0$$

$$\text{or } \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 5 \\ 1 & -1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 30 \\ 50 \\ 0 \end{bmatrix}$$

Where x_1 represents the rope tension $T_1(N)$, x_2 represents the rope tension $T_2(N)$ and x_3 represents the acceleration of the object $a(m/s^2)$. Using Cramer's rule according to equation (3), we get the

$$\text{rope tension, rope tension and object acceleration. } T_1 = \frac{\det A_1}{\det A} = \frac{\begin{vmatrix} 30 & 0 & -3 \\ 50 & 1 & 5 \\ 0 & -1 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & -3 \\ 0 & 1 & 5 \\ 1 & -1 & -2 \end{vmatrix}} = \frac{400}{10} = 40 \text{ N}; T_2 =$$

$$\frac{\det A_2}{\det A} = \frac{\begin{vmatrix} 1 & 30 & -3 \\ 0 & 50 & 5 \\ 1 & 0 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & -3 \\ 0 & 1 & 5 \\ 1 & -1 & -2 \end{vmatrix}} = \frac{360}{10} = 36 \text{ N}; a = \frac{\det A_3}{\det A} = \frac{\begin{vmatrix} 1 & 0 & 30 \\ 0 & 1 & 50 \\ 1 & -1 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & -3 \\ 0 & 1 & 5 \\ 1 & -1 & -2 \end{vmatrix}} = \frac{20}{10} = 2 \text{ m/s}^2$$

Where A_1 is the matrix A with the first column replaced by the coefficient matrix, A_2 is the matrix A with the second column replaced by the coefficient matrix, and A_3 is the matrix A with the third column replaced by the coefficient matrix $\begin{bmatrix} 30 \\ 50 \\ 0 \end{bmatrix}$.

3.2. Results of Normality and Homogeneity Tests

Homogeneity test and normality test are used to determine the sample used in the study. After the homogeneity test was conducted, the sample in this study was 30 students and 30 students who had been tested using SPSS and showed that the sample used was homogeneous. The following is Table 5 regarding the results of the homogeneity test.

Table 5. Homogeneity Test Results

		Levene Statistic	df1	df2	Sig.
Student Grade	Based on Mean	2,855	1	58	,096
	Based on Median	2,167	1	58	,146
	Based on Median and with adjusted df	2,167	1	52,525	,147
	Based on trimmed mean	2,811	1	59	,099

Before analyzing the differences in mathematical thinking ability scores, a normality test was conducted on the mathematical thinking ability scores. The following are the results of the normality test shown in Table 6 below.

Table 6. Normality Test Results

	Kolmogorov-Sminova			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Student Pretest Score	,119	30	,200*	,972	30	,594
Student Posttest Score	,098	30	,200*	,972	30	,783

The results of the normality test in table 6 show a significance value (Sig.) of the pretest value of $0.059 > 0.05$ and a posttest value of $0.079 > 0.05$. So it can be concluded that the pretest and posttest values are normally distributed.

3.3. Mathematical Thinking Ability

The results and discussion of students' mathematical thinking skills after using Cramer's rule in solving pulley LES are seen from the pretest and posttest answers for each indicator, namely: (1) Specializing; (2) Generalizing; (3) Conjecturing; and (4) Convincing. Based on the results of the data analysis, Cramer's rule is effective in solving fixed pulley problems. This is shown through the average results of the N-Gain analysis, namely 0.7, which is included in the high N-Gain category. The following are Table 7 and Table 8 regarding the frequency of students in mathematical thinking skills:

Table 7. Frequency of students at SMAN Jember

Category	Specializing	Generalizing	Conjecture	Convincing
High ($g \geq 0.7$)	28	25	3	-
Moderate ($0.3 \leq g < 0.7$)	2	5	20	30
Low ($g < 0.3$)	-	-	7	-

Table 8. Frequency of students in the Physics Education Study Program

Category	Specializing	Generalizing	Conjecture	Convincing
High ($g \geq 0.7$)	23	22	18	18
Moderate ($0.3 \leq g < 0.7$)	4	5	11	12
Low ($g < 0.3$)	3	3	1	-

The mathematical thinking ability of students in solving fixed pulley problems using Cramer's rule was analyzed using N-Gain analysis. This study measures mathematical thinking ability with four indicators: (1) Specializing; (2) Generalizing; (3) Conjecturing; and (4) Convincing. The following is a graph of the N-Gain Score for each indicator in solving fixed pulley problems using Cramer's rule.

The results of the N-Gain Score for the Specializing, Generalizing, Conjecturing and Convincing indicators for SMAN Jember students and Physics Education study program students are presented in Figure 2 and Figure 3 as follows:

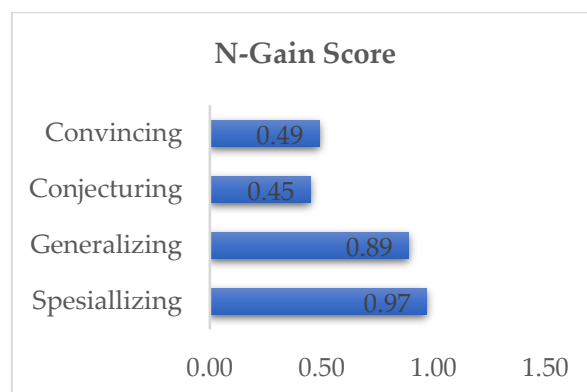


Figure 2. Results of N-Gain Analysis of Each Indicator of SMAN Jember Students

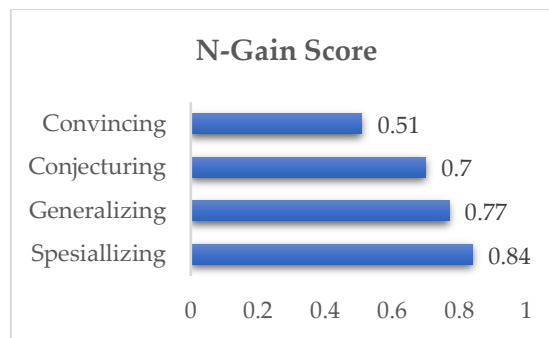


Figure 3. Results of N-Gain Analysis of Each Indicator of Physics Education Study Program Students

The results of the analysis showed that the N-Gain score of the Specializing indicator had a score of 0.97 for SMAN Jember students and 0.84 for students of the Physics Education study program; the Generalizing indicator had a score of 0.89 for SMAN Jember students and 0.77 for students of the Physics Education study program; the Conjecturing indicator had a score of 0.45 for SMAN Jember students and 0.7 for students of the Physics Education study program; the Convincing indicator had a score of 0.49 for SMAN Jember students and 0.51 for students of the Physics Education study program. The discussion of each indicator of mathematical thinking ability is presented as follows:

3.3.1. Specializing (Specialize)

The first indicator of mathematical thinking ability, namely Specializing or the ability of students to identify problems by recording the information that has been given and the questions that need to be answered in the problem (Naja & Sao., 2024). Based on the results of the N-Gain analysis, the increase in mathematical thinking ability in the specializing indicator is high for both students and students as shown in Figure 2 and Figure 3. Writing down known quantities and writing down the problems asked are very important in solving physics problems. And this stage can affect the steps taken next. An example of mathematical thinking ability in the Specializing indicator in solving problems on a fixed pulley during the pretest is shown in Figure 4 and during the posttest is shown in Figure 5

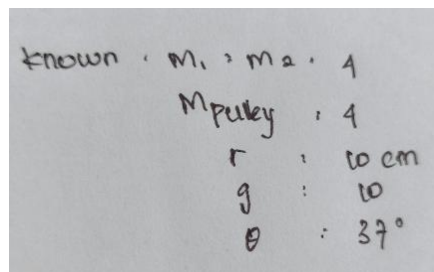


Figure 4. tudent P1's answer

Based on the answer of P1 in Figure 4, students still have not fulfilled the specialization indicator properly. Because most of the students' answers still do not include complete information on the quantities and units that are known and asked. Meanwhile, the answer to P2, as shown in Figure 5.

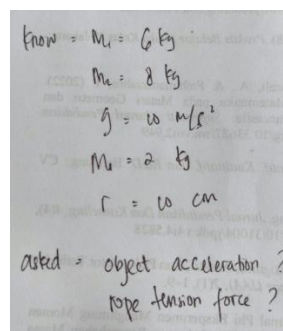


Figure 5. Answers from student P2

Most students were able to identify the information contained in the fixed pulley problem completely, such as the value and units of the first load, m_1 , the load of the two pulley masses and their radii M and m_2 , and gravitational acceleration g correctly. Students also record information related to the problems raised in the questions.

3.3.2. Generalizing (Generalizing)

The second indicator of mathematical thinking ability is Generalizing or generalizing an aspect. Mathematical Thinking Ability in the Generalizing indicator is the ability of students to solve general mathematical equations correctly (Ellis et al., 2021). Based on the results of the N-Gain analysis, the increase in mathematical thinking ability in the Generalizing indicator is high for both students and students as shown in Figure 2 and Figure 3. In this indicator, students are required to be able to draw their free force diagrams, apply Newton's 2nd law to the free force diagram and write linear equations for each object in the pulley system. An example of mathematical thinking ability in the Generalizing indicator in solving problems on a fixed pulley during the pretest is shown in Figure 6 and during the posttest is shown in Figure 7.

Review Object 1

$$\sum F = m \cdot a$$

$$T_1 - W_1 = m \cdot a$$

$$T_1 - M \cdot g = m \cdot a$$

$$T_1 - 6 \times 10 = 6 \cdot a$$

$$T_1 - 6a = 60$$

Review Object 2

$$\sum F = m \cdot a$$

$$W_2 - T_2 = m \cdot a$$

$$m_2 \cdot g - T_2 = m \cdot a$$

$$8 \times 10 - T_2 = 8a$$

$$T_2 + 8a = 80$$

Figure 6. Student P1's answer

Figure 6 shows that students are able to apply Newton's 2nd law to the load in the pulley system but most students do not draw the free force diagram. The direction of the forces acting on the pulley system is directly depicted in the problem image so that most students' answers only get 2 linear equations in the pulley system where the pulley mass is known. Meanwhile, the answer to P2, as shown in Figure 7.

Object 1 = $\sum F_1 = m_1 a$

$$T_1 - m_1 g = m_1 a$$

$$T_1 - 2 \cdot 10 = 2a$$

$$T_1 - 20 = 2a$$

$$T_1 - 2a = 20 \dots (1)$$

Object 2 = $\sum F_2 = m_2 a$

$$W_2 - T_2 = m_2 a$$

$$m_2 \cdot g - T_2 = m_2 a$$

$$3 \cdot 10 - T_2 = 3a$$

$$T_2 + 3a = 30 \dots (2)$$

Pulley = $T_2 - T_1 = b m a$

$$T_2 - T_1 - b m a = 0$$

$$T_1 - T_2 - 3a = 0 \dots (3)$$

Figure 7. Student P2's answers

Figure 7 shows that most students can correctly describe the free force diagram along with the direction of motion and can apply Newton's laws to each object in the free force diagram. With the help of the free force diagram and Newton's 2nd law, 3 linear equations are obtained according to the pulley system problem.

3.3.3. Conjecture (Guess)

The third indicator of mathematical thinking ability is conjecturing or making assumptions through information contained in the questions related to formulas and solution strategies (Masuda et al., 2021). Based on the results of the N-Gain analysis, the increase in mathematical thinking ability in the Conjecturing indicator is 0.45 in the medium category for high school students as shown in Figure 2 and 0.7 in the high category for college students as shown in Figure 3. In this indicator, students are required to be able to solve the Linear Equation System (SLS) to determine the

magnitude of the rope tension and the acceleration of the pulley system using mathematical methods. An example of mathematical thinking ability in the Conjecturing indicator in solving SLS using the substitution and elimination methods during the pretest is shown in Figure 8 and using Cramer's rule during the posttest is shown in Figure 9.

$T - W = M \cdot a + W$ $\sum T = T_1 + T_2 + T$
 $T - 0 = 2 \cdot 0,016 + 0$ $= 0,032 + 0,128 + 0,032$
 $T - 0 = 0,032 + 0$ $= 0,258$
 $T = 0,032$

Figure 8. Student P1's answer

Based on Figure 8, students generally still have difficulty in completing LES by substitution and elimination, although some students can complete it but incompletely. Students' difficulty in completing LES with the substitution and elimination method is because the process is tiered and multilevel and takes a long time. This is in accordance with the research results of Ilme et al., (2023) which states that the substitution and elimination methods are less effective in completing LES with more than 2 variables because they are not time efficient and have a high error rate. Meanwhile, in Figure 9.

$\Rightarrow T_2 = \frac{\det A_2}{\det A} = \frac{\begin{vmatrix} 0 & 1 & -5 & 0 & 1 \\ 10 & 0 & 4 & 40 & 0 \\ 0 & 7 & -1 & 0 & -2 \\ 0 & 1 & -5 & 10 & 1 \\ 1 & 0 & 4 & 1 & 0 \end{vmatrix}}{\begin{vmatrix} 0 & 1 & -5 & 0 & 1 \\ 1 & 0 & 4 & 1 & 0 \\ 1 & 0 & 4 & 1 & 0 \\ 1 & 0 & 4 & 1 & 0 \\ 1 & 0 & 4 & 1 & 0 \end{vmatrix}} = \frac{200 - (-40)}{9 - (-1)} = \frac{240}{10} = 24 \text{ N}$
 $T_1 = \frac{\det A_1}{\det A} = \frac{\begin{vmatrix} 0 & 0 & -5 & 0 & 0 \\ 1 & 0 & 4 & 1 & 0 \\ 1 & 0 & 4 & 1 & 0 \\ 1 & 0 & 4 & 1 & 0 \\ 1 & 0 & 4 & 1 & 0 \end{vmatrix}}{\begin{vmatrix} 0 & 1 & -5 & 0 & 1 \\ 1 & 0 & 4 & 1 & 0 \\ 1 & 0 & 4 & 1 & 0 \\ 1 & 0 & 4 & 1 & 0 \\ 1 & 0 & 4 & 1 & 0 \end{vmatrix}} = \frac{0 - (-200)}{9 - (-1)} = \frac{200}{10} = 20 \text{ N}$
 $a = \frac{\det A_3}{\det A} = \frac{\begin{vmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 4 & 1 & 0 \\ 1 & 0 & 4 & 1 & 0 \\ 1 & 0 & 4 & 1 & 0 \\ 1 & 0 & 4 & 1 & 0 \end{vmatrix}}{\begin{vmatrix} 0 & 1 & -5 & 0 & 1 \\ 1 & 0 & 4 & 1 & 0 \\ 1 & 0 & 4 & 1 & 0 \\ 1 & 0 & 4 & 1 & 0 \\ 1 & 0 & 4 & 1 & 0 \end{vmatrix}} = \frac{10}{10} = 1 \text{ m/s}^2$

Figure 9. Student P2's answers

Students can generally solve LES with the help of Cramer's rule because the process is simpler and not repetitive. So that the increase in mathematical thinking skills in this indicator is in the high category for students and the medium category for high school students. This is because in solving LES using Cramer's rule, students are required to have the ability to determine matrix determinants. Matrix determinants are mathematics lessons that are taught separately from physics lessons for high school students, while for physics education students in the first year they get basic mathematics courses which include matrix material.

3.3.4. Convincing (Convincing)

The last indicator in this study is the convincing process. Wardhani et al. (2016) stated that in essence, high-ability students are able to solve the problems given well so that they get the right solution. In this indicator, students who meet the convincing indicator are able to provide final answers accompanied by meaningful conclusions and complete units. Based on the results of the N-Gain analysis, the increase in mathematical thinking skills in the Convincing indicator is moderate as shown in Figure 2 and Figure 3. An example of mathematical thinking skills in the Convincing indicator is shown in Figure 10 during the pretest and Figure 11 during the posttest.

$a = \frac{78,4}{17} = 4,61 \text{ m/s}^2$
 $T_1 = m_1 a = 16 \cdot 4,61 = 73,76 \text{ N}$
 $T_2 = m_2 a = \frac{1}{2} m a = 73,76 + 4,61 = 78,37 \text{ N}$

Figure 10. Student P1's answer

From Figure 10, most of the students wrote down the acceleration and tension values of the rope that were asked, although most of the values written were not quite right because there were errors when the students carried out the process on the Generalizing indicator which was incomplete. And from the answer of P1, it can be seen that the students did not provide meaningful conclusions or emphasis on the results of their solutions. Meanwhile the answer to P2, as shown in Figure 11.

$$\begin{aligned}
 S_0, T_1 &= \frac{\det A_2}{\det A} = \frac{110.4}{25} \\
 &= 4.416 \text{ N} \\
 T_2 &= \frac{\det A_1}{\det A} = \frac{112.8}{25} \\
 &= 4.512 \text{ N} \\
 a &= \frac{\det A_3}{\det A} = \frac{0.64}{25} \\
 &= 0.0256 \text{ m/s}^2
 \end{aligned}$$

Figure 11. Student P2's answers

Figure 11 shows that most of the students' answers are correct both when determining the rope tension and its acceleration and are also equipped with the correct units. And only a small number of students' answers emphasize their answers so that the value achieved on this indicator cannot be maximized. This is because students feel that by getting the value of rope tension and acceleration, they consider that they have completed the problem being asked.

Based on the results and discussions on each indicator, solving LES using Cramer's rule resulted in an increase in students' mathematical thinking skills in the high category on the specializing and generalizing indicators. Meanwhile, the increase in mathematical thinking skills on the conjecturing indicator is in the medium category for the high school student sample and the high category for the college student sample. This difference is due to differences in students' abilities in determining the determinant of the 3rd order matrix. And for the convicting indicator, the increase in students' mathematical thinking skills is in the medium category for both samples because most students do not emphasize the results of their solutions. So Cramer's rule is quite effective for solving LES on a fixed pulley system. This is in accordance with the results of the study by Sari et al (2021) which stated that the mathematical thinking skills on the specializing indicator are quite good but for the other three indicators they are still low. The less than optimal mathematical thinking skills on the third and fourth indicators are because students generally do not complete mathematical operations in solving LES using either the elimination method or Cramer's rule and do not draw conclusions from the results of their calculations. Tupulu et al, (2023) stated that students' mathematical thinking skills are still quite low because students do not complete the problems.

3.4. Learning outcomes

The effectiveness of Cramer's rule in solving LES on fixed pulley material is not only seen from the increase in mathematical thinking skills but also seen from the increase in learning outcomes. Figure 9 shows that there was an increase in the lowest, highest and average scores of students after students used Cramer's rule in solving LES on the pulley system.

Table 9. Physics Learning Outcome Data

Information	Mark		
	Pretest	Posttest	N-Gain
Number of Students	30	30	0.4
Lowest Value	9	48	(Currently)
The highest score	53	75	
Average	30.8	58.8	

Based on the N-Gain analysis, the increase in students' learning outcomes was 0.4 in the moderate category. The increase in students' learning outcomes was due to the ability to think mathematically for all indicators (Specializing, Generalizing, Conjecturing, and Convicing) increasing after students used Cramer's rule in solving LES on the pulley system. Mathematical thinking skills

are very much needed in solving physics problems, especially problems expressed in the form of equations. Neumann et al., (2021) stated that solving physics problems requires mathematical skills, even mathematics is seen as the language of physics that has a function to formulate problems in mathematical models. This statement is also supported by Woitkowski (2020) who stated that mathematical skills are important in the physics learning process.

Students' mathematical thinking skills can affect their learning outcomes. This is also supported by the research results of Genc & Erbas (2019) and Ulya & Rahayu (2021) which concluded that students' mathematical thinking skills greatly affect their mindset and learning outcomes. Improving students' mathematical thinking skills can improve learning outcomes (Szabo et al, 2020). Saraswati et al (2019) also stated that mathematical skills affect physics learning outcomes. Based on the results above, it can be said that the application of Cramer's rule in solving LES on fixed pulley material can improve students' mathematical thinking skills so that it has an impact on improving their learning outcomes. This is in accordance with the research results of Ayu et al., (2024) which concluded that the use of matrices can be used as an alternative method in solving LES. The effectiveness of Cramer's rule in LES is also supported by the research results of Ndlovu & Brijalli (2019) which stated that Cramer's rule is an effective method in linear equations to significantly improve learning outcomes.

3.5. Response

The results of the response analysis in this study where students were given treatment of applying Cramer's rule in solving LES on the Fixed Pulley material. The analysis of response data in this study used the Likert Scale. The results of the response analysis are presented in Table 10.

Table 10. Response Analysis Results

Indicator	Presentation	Category	Average/ Category
Interest	81%	Very Positive	81%
Motivation	80%	Very Positive	Very Positive
Interest	80%	Very Positive	
Satisfaction	82%	Very Positive	
Response	82%	Very Positive	

Based on the analysis results shown in Table 10, it can be concluded that the students' responses in applying Cramer's rule assisted by Sarrus in solving LES on the Fixed Pulley material showed a very positive response with an average percentage of 81%.

3.6. Interest Indicator

The first response indicator in this study is interest. This interest indicator received a student response that was included in the very positive category with a percentage of 81%. An innovative learning process can invite students to have an interest in implementing the learning process (Sari & Lestari, 2021). This shows that students are interested, enthusiastic and happy in learning to use Cramer's rule in solving LES on the Fixed Pulley material.

3.7. Motivation Indicators

The second response indicator in this study is motivation. Motivation received a very positive response from students with a percentage of 80%. This shows that students are very motivated, enthusiastic, and confident in using Cramer's rule in working on fixed pulley problems. With students' self-confidence after using Cramer's rule in working on fixed pulley problems, it can motivate students to understand the material thoroughly and complete the tasks given (Humaidi & Raharjo, 2022).

3.8. Interest Indicator

The third response indicator in this study is interest. Interest received a very positive response with an indicator percentage of 80%. This shows that students have an interest in the learning process using Cramer's rule in solving LES on fixed pulley material. In the learning process, students' interest in learning will affect students' learning concentration so that innovative learning is needed (Humaidi & Raharjo, 2022).

3.9. Satisfaction Indicators

The fourth response indicator in this study is satisfaction. This satisfaction indicator received a response from students that was included in the very positive category with a percentage of 82%. This proves that Cramer's rule not only makes it easier to solve problems, but also broadens horizons, encourages exploration in fixed pulley problems, and meets the needs and expectations of students.

3.10. Response Indicator

The fifth response indicator in this study is the response. This response indicator received a student response that was included in the very positive category with a percentage of 82%. This shows that students' use of Cramer's rule assisted by Sarrus in solving LES on fixed pulley material is an alternative new solution in solving so that it can improve students' mathematical thinking skills. This also shows that Cramer's rule assisted by Sarrus has easy-to-understand stages of solution so that problem solving is more concise.

The responses shown are included in the very high category which shows that Cramer's rule is more efficient in solving LES problems in fixed pulley material. Based on the results of the response analysis, it was shown that students were more enthusiastic in learning. In line with the research of Chen et al, (2023) that mathematical methods can increase students' desire to learn and gain pleasure and self-confidence. Research by Krisnawanto et al, (2024) stated that the application of Cramer's rule assisted by Sarrus received a positive response from students and was proven to be effective as an alternative to solving physics problems. Supported by research by Mavron & Phillips. (2023) which stated that Cramer's rule is more effective in terms of linear equations, so that the analysis of linear equations is simpler using matrix algebra.

4. Conclusion

Based on the results and discussions above, the application of Cramer's rule is proven to be effective as an alternative method in solving LES on pulley material. This is proven by the application of Cramer's rule in solving LES on pulley material can still improve mathematical thinking skills, student learning outcomes. In addition, the application of Cramer's rule received a very positive response from students for indicators of motivation, interest, satisfaction and response. The results of the study showed that the increase in mathematical thinking skills was in line with the increase in student learning outcomes. This is in accordance with the results of previous studies which stated that mathematical thinking skills can affect learning outcomes. Based on the results of this study, Cramer's rule can be one of the alternative mathematical methods besides the substitution-elimination method in solving LES on physics problems and can be introduced in the learning process.

Although Cramer's rule has proven effective in solving LES on fixed pulley material because it can improve mathematical thinking skills and learning outcomes, further research needs to be conducted using a larger data distribution (sample). In addition, it is also continued by seeing how the application of Cramer's rule affects the application of LES on pulley material or other physics materials on students' mathematical abilities and learning outcomes.

Author Contributions

Bambang Supriadi: Conceptualization, Introduction, Methodology, Validation; Sisilia Nur Hikmah Anggraeni: Introduction, conclusion; Nurul Yunda Nanik Purwanti: Introduction, results, Discussion; Emma Berliana Pujiningtyas: Introduction, Results, Discussion; Dina Mahartika: Introduction, Discussion, References. All authors have equal contributions to the paper. All the authors have read and approved the final manuscript.

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References

- Aida, N., Verawati, N., Pratiwi, I., & Muttaqin, K. (2023). Implementasi Metode Cramer Menggunakan Microsoft Excel Dalam Penyelesaian Rangkaian Listrik Arus Searah. *Jurnal Pembelajaran Matematika Inovatif*, 6(5), 1782. <https://doi.org/10.22460/jpmpi.v6i5.18384>
- Alhasan, Y. A. (2021). Types of system of the neutrosophic linear equations and Cramer's rule. *Neutrosophic Sets and Systems*, 45(1), 26. https://digitalrepository.unm.edu/nss_journal/vol45/iss1/26
- Aminah, S., & Radita, N. (2020). Pengembangan modul pembelajaran aljabar linier dan Matriks dengan pendekatan inkuiri untuk mahasiswa Teknik informatika. *MUST: Journal of Mathematics Education, Science and Technology*, 5(2), 156-170. <https://doi.org/10.30651/must.v5i2.5884>
- Anam, K., & Arnas, Y. (2019). Metoda Cramer Untuk Solusi Analisa Rangkaian Listrik Menggunakan Scilab. *Langit Biru: Jurnal Ilmiah Aviasi*, 12(1), 61-68. <https://journal.ppicurug.ac.id/index.php/jurnal-ilmiah-aviasi/article/view/144>
- Anwar, N. T. (2018). Peran kemampuan literasi matematis pada pembelajaran matematika abad-21. *Prisma*, 1, 364-370. <https://journal.unnes.ac.id/sju/index.php/prisma/>
- Ayu, M. P., Bouk, Y. M., & Popo, A. T. (2024). Analisis Kesalahan Mahasiswa Pada Sistem Persamaan Linear Dengan Operasi Baris Elementer Kelas Aljabar Linear Dan Matriks. *Jurnal Jendela Pendidikan*, 4(03), 301-309. <https://doi.org/10.57008/jjp.v4i03.810>
- Bramasto, S., & Khairiani, D. (2022). Prediksi Daya Output Sistem Pembangkit Listrik Tenaga Surya (PLTS) Menggunakan Regresi Linear Berganda. *Faktor Exacta*, 15(3), 142. <http://dx.doi.org/10.30998/faktorexacta.v15i2.13254>
- Busrah, Z. (2019). Matematika Komputasi Berbasis Pemrograman Matlab. <https://repository.iainpare.ac.id/id/eprint/2702>
- Chen, D., Putri, N. D., Meliza, W., Astuti, Y., Wicaksono, L. Y., & Putri, W. A. (2021). Identifikasi Minat Siswa SMA Kelas X Terhadap Mata Pelajaran Fisika. *PENDIPA Journal of Science Education*, 5(1), 36-39. <https://doi.org/10.33369/pendipa.5.1.36-45>
- Chen, R., Fang, L., Zhang, M., & Kong, J. The Teaching Mode Starting from the Application Example-Take Linear Algebra Courses as an Example. *International Research Journal of Advanced Engineering and Science*. 8(4):181-184.
- Devita, R. (2022). Eksistensi Metode Cramer Sebagai Solusi Penyelesaian SPL Dalam Kasus Rangkaian Listrik. *Jurnal Syntax Fusion*, 2(10), 859-970. <https://doi.org/10.54543/fusion.v2i10.284>
- Drijvers, P., Kodde-Buitenhuis, H., & Doorman, M. (2019). Assessing mathematical thinking as part of curriculum reform in the Netherlands. *Educational studies in mathematics*, 102(3), 435-456. <https://doi.org/10.1007/s10649-019-09905-7>
- Ellis, A. B., Lockwood, E., Tillema, E., & Moore, K. (2022). Generalization across multiple mathematical domains: Relating, forming, and extending. *Cognition and Instruction*, 40(3), 351-384. <https://doi.org/10.1080/07370008.2021.2000989>
- Fajri, M. (2017). Kemampuan berpikir matematis dalam konteks pembelajaran abad 21 di sekolah dasar. *Lemma: Letters of Mathematics Education*, 3(2). <https://doi.org/10.22202/jl.2017.v3i2.1884>
- Ferdianto, F., Sukestiyarno, Y. L., & Widowati, I. J. (2022). Mathematical Thinking Process On Numeracy Literacy Problems For Middle School Students. *Journal of Positive School Psychology*, 6(8), 6909-6923. <http://journalppw.com>
- Genc, M., & Erbas, A. K. (2019). Secondary mathematics teachers' conceptions of mathematical literacy. *International Journal of Education in Mathematics, Science and Technology*, 7(3), 222-237. <https://www.ijemst.net/index.php/ijemst/article/view/611>
- Hake, R. R. (1999). *Analyzing change/gain scores*.
- Hamidah, N., & Setiawan, W. (2019). Analisis minat belajar siswa SMA Kelas XI pada materi matriks. *Journal on Education*, 1(2), 457-463. <https://doi.org/10.31004/joe.v1i2.96>
- Homaeinezhad, M. R., & Ebrahimi, M. M. (2024). Numerical Approach for Nonlinear Dynamics Simulation of Belt-Pulley XY Positioning Mechanism. *Engineering Science & Technology*, 352-373. <https://doi.org/10.37256/est.5220244538>
- Humaidi, H., Qohar, A., & Rahardjo, S. (2022). Respon siswa terhadap penggunaan video youtube sebagai media pembelajaran daring matematika. *JIPM (Jurnal Ilmiah Pendidikan Matematika)*, 10(2), 153-162. DOI : 10.25273/jipm.v10i2.9108
- Ilmi, U., Faroh, R. A., Hanifah, A. I., & Mukhoyyaroh, N. I. (2023). STUDI METODE CRAMER DAN METODE INVERS Matriks PADA PERSOALAN RANGKAIAN LISTRIK. *JE-Unisla*, 8(2), 92-96. <https://doi.org/10.30736/je-unisla.v8i2.1079>
- Kartini, K. S., & Putra, I. N. T. A. (2020). Respon siswa terhadap pengembangan media pembelajaran interaktif berbasis android. *Jurnal Pendidikan Kimia Indonesia*, 4(1), 12-19. <https://doi.org/10.23887/jpk.v4i1.24981>
- Krisnawanto, E. D., Supriadi, B., Dewi, N. M., & Abdillah, U. F. (2024). Implementation of Cramer's rule assisted by the Sarrus method in solving the problem of 2-loop unidirectional electrical circuits. *JIPFRI (Jurnal Inovasi Pendidikan Fisika dan Riset Ilmiah)*, 8(2), 60-66. <https://doi.org/10.30599/jipfri.v8i2.2921>

- Luo, H., Wu, S., & Xie, N. (2021, September). Three methods for solving systems of linear equations: Comparing the advantages and disadvantages. In *Journal of Physics: Conference Series* (Vol. 2012, No. 1, p. 012061). IOP Publishing. DOI 10.1088/1742-6596/2012/1/012061
- Maharani, N. (2020). Perbandingan Tingkat Pemahaman Mahasiswa STMIK STIKOM Indonesia Pada Metoda Sarrus dan Metoda Cramer pada Penyelesaian Sistem Persamaan Linier. *PENDIPA Journal of Science Education*, 4(2), 66-73. <http://dx.doi.org/10.33369/pendipa.4.2.66-73>
- Manly, B. F. (1992). *The design and analysis of research studies*. Cambridge University Press.
- Masuda, A., Pambudi, D. S., & Murtikusuma, R. P. (2021). Analisis Penalaran Matematis Siswa SMA Kelas XI dalam Menyelesaikan Soal Barisan dan Deret Aritmetika Ditinjau dari Gaya Belajar Honey-Mumford. *Jurnal Riset Pendidikan dan Inovasi Pembelajaran Matematika (JRPIPM)*, 5(1), 56-68. <https://doi.org/10.26740/jrpipm.v5n1.p56-68>
- Mavron, V. C., & Phillips, T. N. (2023). *Matrices and Determinants*. In *Elements of Mathematics for Economics and Finance* (pp. 223-264). Cham: Springer International Publishing. https://doi.org/10.1007/978-3-031-43910-0_10
- Naja, F. Y., & Sao, S. (2024). Peningkatan Berpikir Matematis Siswa Sekolah Dasar Melalui Implementasi Model Problem Based Learning. *Jurnal Cendekia: Jurnal Pendidikan Matematika*, 8(2), 1049-1059. <https://doi.org/10.31004/cendekia.v8i2.3124>
- Ndlovu, Z., & Brijlall, D. (2019). Pre-service mathematics teachers' mental constructions when using Cramer's rule. *South African Journal of Education*, 39(1). <http://dx.doi.org/10.15700/saje.v39n1a1550>
- Ndraha, I. S., & Mendrofa, R. N. (2022). Analisis hubungan minat belajar dengan hasil belajar Matematika. *Educativo: Jurnal Pendidikan*, 1(2), 672-681. <https://doi.org/10.56248/educativo.v1i2.92>
- Neumann, I., Jeschke, C., & Heinze, A. (2021). First year students' resilience to cope with mathematics exercises in the university mathematics studies. *Journal für Mathematik-Didaktik*, 42(2), 307-333. <https://doi.org/10.1007/s13138-020-00177-w>
- Norman, E., Paramansyah, A., & Abdan, M. S. (2022). The Role of Organizational Culture in the Effectiveness of School Organizations. *Da'watuna: Journal of Communication and Islamic Broadcasting*, 2(3), 254-269. <https://doi.org/10.47467/dawatuna.v2i3.2059>
- Nurhayati, N., Zuhra, F., & Salehha, O. P. (2021). Penerapan model pembelajaran project based learning berbantuan geogebra untuk meningkatkan hasil belajar siswa. *Jurnal Pendidikan Matematika (Jupitek)*, 4(2), 73-78. <https://doi.org/10.30598/jupitekvol4iss2pp73-78>
- Nurlatipah, N., Juanda, A., & Maryuningsih, Y. (2015). Pengembangan media pembelajaran komik sains yang disertai foto untuk meningkatkan hasil belajar siswa kelas VII SMPN 2 SUMBER pada pokok bahasan ekosistem. *Scientiae Educatia: Jurnal Pendidikan Sains*, 4(2). <https://doi.org/10.21831/jipi.v4i1.21076>
- OECD. (2019). Program for International Students Assessment (PISA), Country Note: Indonesia. https://www.oecd.org/pisa/publications/PISA2018_CN_IDN.pdf diunduh pada Desember 2024.
- Ornek, F., Robinson, W. R., & Haugan, M. P. (2008). What makes physics difficult. *International Journal of Environment & Science Education*, 3(1), 30-34. <https://eric.ed.gov/?id=EJ894842>
- Pinahayu, E. A. R., Adnyani, L. P. W., Mulyani, N., & Sriyono, S. (2023). Analisis Miskonsepsi Mahasiswa dalam Menyelesaikan Soal Sistem Persamaan Linier (SPL) Berdasarkan Tahapan Newman. *Journal on Education*, 5(2), 4381-4390. <https://doi.org/10.31004/joe.v5i2.1157>
- Prastika, Y. D. (2021). Hubungan minat belajar dan hasil belajar pada mata pelajaran matematika di SMK Yadika Bandar Lampung. *Jurnal Ilmiah Matematika Realistik*, 2(1), 26-32. <http://dx.doi.org/10.33365/ji-mr.v1i2.519>
- Pratiwi, Y. D. (2024). *Matriks dan Vektor*. wawasan Ilmu.
- Putra, A. & Heriyanto. (2020). Analysis of student's understanding about Newton's laws, in terms of perceptions to learning in senior high school. *Journal of Physics: Conference Series*, 1481(1), 012134. <https://doi.org/10.1088/1742-6596/1481/1/012134>
- Putri, R. D. R., Ratnasari, T., Trimadani, D., Halimatussakdiah, H., Husna, E. N., & Yulianti, W. (2022). Pentingnya Keterampilan Abad 21 Dalam Pembelajaran Matematika. *Science and Education Journal (SICEDU)*, 1(2), 449-459. <https://www.researchgate.net/publication/385286524>
- Rahayu, E. C., Supriadi, B., & Dewi, N. M. (2023). Aturan Cramer Berbantuan Excel pada Materi Rangkaian Listrik Searah Dua Loop untuk Mengukur Kemampuan Berpikir Komputasi Peserta Didik. *U-Teach: Journal Education of Young Physics Teacher*, 4(2), 95-102. <https://doi.org/10.30599/uteach.v4i2.600>
- Rahayu, R., Iskandar, S., & Abidin, Y. (2022). Inovasi pembelajaran abad 21 dan penerapannya di Indonesia. *Jurnal Basicedu*, jbasic.org, <https://jbasic.org/index.php/basicedu/article/view/2082>
- Rizkita, N. I., & Mufit, F. (2022). Analisis Pemahaman Konsep dan Sikap Siswa Terhadap Belajar Fisika Pada Materi Hukum Newton Tentang Gerak. *Jurnal Eksakta Pendidikan (Jep)*, 6(2), 233-242. <https://doi.org/10.24036/jep.v1i2.50>
- Saraswati, D. L., Lestari, I., Andinny, Y., & Hikmah, N. (2020, February). The effect of basic mathematical abilities on learning outcomes of physics education students. In *Journal of Physics: Conference Series* (Vol. 1464, No. 1, p. 012008). IOP Publishing. <http://dx.doi.org/10.1088/1742-6596/1464/1/012008>
- Sari, E. Y. (2015). Pengaruh Pola Asuh Orang Tua Terhadap Prestasi Belajar IPS Siswa Kelas V SD Se-Gugus III Seyegan Sleman Tahun Ajaran 2014/2015. *Universitas PGRI Yogyakarta*. <http://repository.upy.ac.id/id/eprint/197>

- Sari, P., Dwikoranto, D., & Lestari, N. A. (2021). Analisis respon dan ketertarikan peserta didik terhadap pelaksanaan pembelajaran fisika berbasis environmental learning di SMA. *PENDIPA Journal of Science Education*, 5(3), 337-344. <https://doi.org/10.33369/pendipa.5.3.337-344>
- Sari, W., Nasriadi, A., & Salmina, M. (2021). Analisis kemampuan berpikir matematis siswa menyelesaikan soal ujian akhir semester (Uas) pada tahun ajaran 2020 di SMAN 1 Teluk dalam Kabupaten Simeulue. *Jurnal Ilmiah Mahasiswa Pendidikan*, 2(1).
- Siagan, M. V., Saragih, S., & Sinaga, B. (2019). Development of Learning Materials Oriented on Problem-Based Learning Model to Improve Students' Mathematical Problem Solving Ability and Metacognition Ability. *International electronic journal of mathematics education*, 14(2), 331-340. <https://doi.org/10.29333/iejme/5717>
- Sihombing, A. A., Anugrahsari, S., Parlina, N., & Kusumastuti, Y. S. (2021). "Merdeka Belajar" in an Online Learning during the COVID-19 Outbreak: Concept and Implementation. *Asian Journal of University Education*, 17(4), 35-45. <https://doi.org/10.24191/ajue.v17i4.16207>
- Simamora, N. N. (2022). *Pengembangan E-Modul Fisika Matematika I Menggunakan Flip PDF Professional Pada Materi Operasi Matriks* (Doctoral dissertation, UNIVERSITAS JAMBI). <https://repository.unja.ac.id/>
- Siombone, S. H., & Niwele, A. (2023). Studi Korelasi Kemampuan Awal Matematika Mahasiswa dengan Pencapaian Kognitif Fisika Umum Konsep Gerak Peluru pada Tingkatan Berpikir Aplikasi (C3) DAN ANALISIS (C4). *Jurnal Pendidikan Fisika*, 12(2), 116. <https://doi.org/10.24114/jpf.v12i2.49418>
- Stacey, Kaye. 2010. *Tinking Mathematically: Second Edition. England: Pearson Educion*
- Sukmayadi, V., & Yahya, A. (2020). Indonesian education landscape and the 21st century challenges. *Journal of Social Studies Education Research*, 11(4), 219-234. <https://www.learntechlib.org/p/218538/>
- Sulistiyorini, S., & Listiadi, A. (2022). Pengembangan media pembelajaran Ispring Suite 10 berbasis android pada materi jurnal penyesuaian di SMK. *Edukatif: Jurnal Ilmu Pendidikan*, 4(2), 2116-2126. <https://edukatif.org/index.php/edukatif/article/view/2288>
- Szabo, Z. K., Kórtesi, P., Guncaga, J., Szabo, D., & Neag, R. (2020). Examples of problem-solving strategies in mathematics education supporting the sustainability of 21st-century skills. *Sustainability*, 12(23), 10113. <https://doi.org/10.3390/su122310113>
- Taufik, A. (2024). Meningkatkan Keterampilan Hots dan Hasil Belajar Matematika Siswa Melalui Media Kartu Soal Dalam Problem Based Learning. *Indonesian Journal of Educational Science (IJES)*, 6(2), 106-119. <https://ojs.unsulbar.ac.id/>
- Tupulu, N, Jamiah, Y, Rustam, R, & ... (2023). Pengembangan Kemampuan Berpikir Matematis untuk Penguatan Disposisi Matematis Melalui Kolaborasi antara Siswa dan Guru. ... *Matematika*, e-journal.undikma.ac.id, <https://e-journal.undikma.ac.id/index.php/jmpm/article/view/7853>
- Ulya, H., & Rahayu, R. (2021). Hubungan Keterampilan Proses Berpikir Matematis dengan Hasil Belajar Mahasiswa. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, 10(1), 262-272. <https://doi.org/10.24127/ajpm.v10i1.3361>
- Wahyudianti, R. (2023). Penerapan Metode Minor-Kofaktor Matrik Bujursangkar Orde-3 dalam Aturan Cramer pada Penyelesaian Soal Rangkaian Listrik DC 2-Loop. *Navigation Physics: Journal of Physics Education*, 5(2), 37-45. <https://doi.org/10.30998/npjpe.v5i2.2326>
- Wardhani, W. A., Subanji, S., & Dwiwana, D. (2016). Proses berpikir siswa berdasarkan kerangka kerja mason. *Jurnal Pendidikan: Teori, Penelitian, dan Pengembangan*, 1(3), 297-313. <http://dx.doi.org/10.17977/jp.v1i3.6152>
- Woitkowski, D. (2020). Tracing physics content knowledge gains using content complexity levels. *International Journal of Science Education*, 42(10), 1585-1608. <https://doi.org/10.1080/09500693.2020.1772520>
- Yudhoyono, T, R, M, K, Huda., dan U,Q, Ain. Pengaruh model problem based learning terhadap kemampuan literasi numerasi siswa pada mata pelajaran matematika. (2024). *Jurnal Pendidikan dan Pengajaran*. 2 (9): 637-648. <https://jurnal.kolibi.org/index.php/cendikia/article/view/2726>