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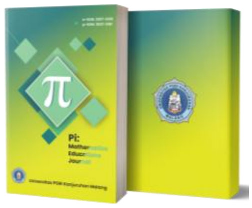

Analysis of Students' Errors in Solving Mathematical Proof Problems on Real Numbers

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ABSTRACT
The aim of this study is to analyze students' errors in solving mathematical proof problems on real numbers using Kastolan's Theory. This study used a qualitative descriptive approach with third-semester students, who have taken the Real Analysis course, as the subjects. Data was collected through validated written tests consisting of three proof questions and interviews. Data analysis used the Miles and Huberman model with criterion sampling for subject selection. The results of the study showed three types of errors, namely conceptual, procedural, and technical errors. The conceptual errors found were errors in using definitions and errors in using arguments caused by a lack of understanding of deductive proofs. The procedural errors found were errors in writing systematic steps and errors caused by answers that did not return to the conclusion due to difficulties in constructing arguments systematically. Meanwhile, the technical errors found were errors in writing notation and errors in copying simple expressions due to carelessness. The most dominant errors were technical errors, especially in writing mathematical notation or symbols and copying or simplifying expressions. Further research is recommended to develop and test effective learning strategies or models to minimise conceptual, procedural, and technical errors.

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1. Introduction

Real analysis is one of the mandatory courses in mathematics education. This course aims to develop students' higher-order thinking skills, as it involves analytical, logical, critical, and systematic processes. Proofs play a central role in real analysis course. Solving proof problems in real analysis is closely related to the existing definitions, consequences, theorems, or lemmas. Therefore, how students systematically wrote their reasoning is considered an important aspect in real analysis course.

Mathematical proof is a deductive reasoning process that shows the truth of a mathematical statement by referring to previously established axioms, definitions, and theorems [1]. Proofs are fundamental, essential, and inseparable in mathematics [2]. Mathematical proof is a way to validate the truth of a statement, that becomes the foundation for other sciences.

Mathematical proof ability refers to the skill in writing arguments logically and systematically to show the truth of a statement. Mathematical proof ability also encompasses the ability to understand both symbols and statements along with their reasoning or evidence [3]. In mathematical proof, students must have the ability in giving arguments, formulating and examining assumptions, applying several proving and understanding methods, and evaluating mathematical arguments and proofs.

Several former researches have shown that students still face difficulties in mathematical proof. Former research indicates that misconceptions in real analysis frequently occur among students, causing challenges in providing proofs [2]. Another research reveals that the students are less proficient and not accustomized to following correct and well-organized procedures in answering proof questions [4]. Students also made errors in solving problems involving real number bi-implications. These difficulties in working on real analysis are indicated by errors made when solving problems [5].

Error analysis is employed to examine the errors made by students during the process of solving problems and more detail tasks [4]. Errors are deviations from something that is systematic, consistent, and incidental [6]. The purpose of error analysis is to find appropriate solutions to overcome the errors made [5]. There are several theories for analyzing errors, Newman's Theory and Polya's Theory can be conducted in word problems [7]. Meanwhile Kastolan's Theory can be used to analyze students' errors in mathematical proof [8]. Hence, the theory used in this study is Kastolan's Theory.

Error analysis based on Kastolan's Theory is a technique used to examine students' errors in solving mathematical problems [9]. Kastolan classifies errors into three types: conceptual errors, procedural errors, and technical errors [10][11]. This aligns with Radatz's statement that students' errors are failures of student in understanding certain concepts, techniques, and problems [12].

Conceptual errors are mistakes made by students in applying concepts that exists in real numbers material. Procedural errors are mistakes in using rules, formulas, or principle in mathematical proofs on real numbers. Operational errors are mistakes made during the execution of operations used in mathematical proof on real numbers [13].

Previous studies have analyzed students' errors in real analysis course on bi-implication proof problems using Kastolan's error analysis [14]. Error analysis has also been conducted by other researchers on real analysis problems involving absolute value, identifying conceptual and procedural errors in students' answer [6]. Meanwhile, another study on real analysis course topics of completeness axioms and limits of real numbers sequences, reveales students' error using Newmann's Error Analysis (NEA), which includes reading errors, comprehension errors, transformation errors, and process errors [15].

Based on the description above, we chose to study "Analysis of Students' Errors in Solving Mathematical Proof Problems on Real Numbers". We conducted this study using Kastolan's errors theory on real numbers topics, including field axioms, order axioms, absolute value definitions and theorems in real numbers.

2. Methods

This study employs qualitative descriptive research. Qualititative descriptive research is a study that describe phenomena based on certain categories, using written words from the observed subjects [5]. Descriptive research refers to activities that gather information from

events to achieve objectives through data collection and report preparation [16]. Descriptive research depicts actual conditions without manipulating and changing variables [14].
What we do in this study to obtain qualitative data is asked students to complete written real number proof problems. While the students were completing the problems, the researcher observed how they carried out the proofs. During the observation, the researcher noted any student behavior that indicated conceptual, procedural, or technical errors. The researcher conducted semi-structured interviews to find out the reasoning behind each step of the proof. In addition, to identify the cases that occured, an analyze was carried out. The results then presented as descriptive form from errors in mathematical proof on real numbers.

This study was conducted on third-semester students of the 2025/2026 academic year who had taken the real analysis course. Previously, students had taken Introduction to Basic Mathematics and Calculus, which are prerequisites for Real Analysis. The third semester is a "transition to proof" semester, in which students shift from solving problems to constructing proofs. Only the third semester was selected in order to maintain the homogeneity of the research subjects [17]. The selection of subjects to be described used criterion sampling, which involved selecting cases relevant to the description of mistakes made by students, namely students with conceptual errors, students with procedural errors, students with technical errors, and students with good communication skills.

The research began with the identification of problems related to student errors in mathematical proofs. Next, the research is based on a theoretical study that includes error analysis, mathematical proofs, and Kastolan's Theory, then the research objectives are formulated to identify and describe student errors based on this theory. The research process continues with data collection through tests and interviews, data analysis using Kastolan's Error Theory and the Miles & Huberman analysis model, and finally produces research conclusions. Procedures conducted in this study are illustated by flowchart shown in Figure 1.

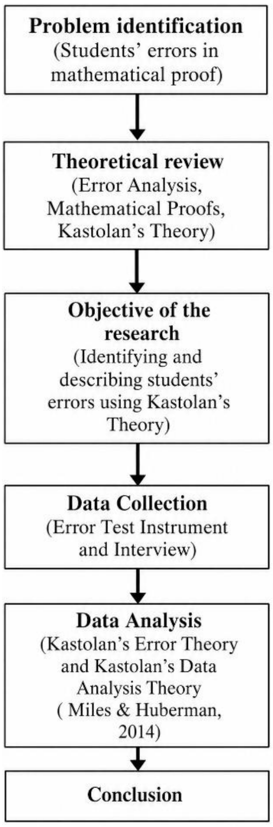

Figure 1. Research Stages

The data collection technique involved using a test as the instrument. The test consisted of three essay problems about proofs related to algebraic properties, order properties, and absolute value on real numbers. The problems given were adopted from Bartle & Sherbert. The questions were selected and adjusted based on error indicators according to Kastolan's classification to ensure that each item was representative of each error category. After the instrument was developed, it was validated to ensure its suitability for the research objectives and the ability indicators being measured. The validation was carried out by two mathematics education lecturers who had taught real analysis.

While the indicators, descriptions, and problems used are explained in Table 1 as follows.

Table 1. Indicators, Descriptions, and Problems

No	Indicator	Description	Problem
1	Students are able to prove statements using the field axioms of real numbers	This problem is used to examine the ability of students in mathematical proof involving the multiplicative field axioms of real numbers (multiplicative identity, inverse, commutative property, and associative property)	If $a \neq 0$ and $b \neq 0$, prove that $\frac{1}{ab} = \frac{1}{a} \cdot \frac{1}{b}$
2	Students are able to prove statements using the order axiom of real numbers	This problem is used to examine the ability of students in mathematical proof involving the order axioms of real numbers	If $a \leq b$ and $c < d$, prove that $a + c < b + d$
3	Students are able to prove statements using the definiton of absolute value	The problem is used to examine the ability of students in mathematical proof involving the definition of absolute value	If $a, b \in \mathbb{R}$ and $b \neq 0$ prove that $\left\|\frac{a}{b}\right\| = \frac{\|a\|}{\|b\|}$

[18]

The Kastolan's error indicators used in this study are shown in Table 2.

Table 2. Kastolan's Error Types and Indicators

No	Error Type	Error Indicators
1	Conceptual Errors	a. Errors in using properties or theorems b. Errors in constructing logical arguments
2	Procedural Errors	a. Errors in performing steps using proof methods or approaches b. The solution does not lead back to the conclusion being proved
3	Technical Errors	a. Errors in writing the mathematical notation or symbols b. Errors in copying or simplifying expressions

(Adaptation: [19])

The data analysis technique in this study used the procedures of Miles & Huberman. The analysis technique applied are as follows [20]:

1. Data collection was conducted using an instrument in the form of analysis problems.

2. Data reduction was carried out through analysis of the answers, identify errors in students' answers and classify errors based on error categories according to Kastolan's Theory. Each error category is coded as conceptual error (C), procedural error (P), and technical error (T) with reference to the rubric that has been compiled.
3. Data display, the reduced data were then presented as an error percentage table according to Kastolan's Theory. The error percentage was calculated by dividing the number of students who made errors in each type of error by the total number of students. The researchers then described the errors in each category from the subjects' answers that were representative of errors based on Kastolan's errors.
4. Conclusion drawing involving concluding the results of the analysis of students' errors in solving mathematical proof problems on real numbers according to Kastolan's Error Theory. This step aims to obtain a clearer overview of the analysis results.

3. Result and Discussion

Before being used in research, the instrument was validated by two mathematics education lecturers who had taught Real Analysis. The validators' assessment results obtained an average score of 3.62 on a scale of 4, with a category of highly suitable. The validators' suggestion was that it was necessary to develop error indicators to distinguish between conceptual errors, procedural errors, and technical errors. In addition, for the questions, there needed to be an emphasis on the use of formal definitions in the proofs, but this input did not change the substance and structure of the questions, so the instrument could be used in research.

Based on the results of the analysis of students' answers in mathematical proofs according to the Kastolan's error indicators, the following data in Table 3 were obtained:

Table 3. Percentage of Errors in Answers According to Kastolan

Error According to Kastolan	Error Type	Percentage (%) of Errors Problem Number		
		1	2	3
Conceptual Errors (C)	1. Errors in using definition, properties or theorems			11
	2. Errors in constructing logical arguments		11	
Procedural Errors (P)	1. Errors in performing steps using proof methods or approaches			11
	2. The solution does not lead back to the conclusion being proved	22	11	11
Technical Errors (T)	1. Errors in writing the mathematical notation or symbols	78		
	2. Errors in copying or simplifying expressions	22	22	11

Based on Table 3, Three types of errors were identified. On Problem 1, there were no conceptual errors. Procedural errors occurred in the form of solutions not leading back to the statement being proved (P2), with a percentage of 22% (2 students). Technical errors including mistakes in writing mathematical notation or symbols (T1), with a percentage of 78% (7 students), and errors in copying or simplifiying expressions (T2), with a percentage of 22% (two students).

On Problem 2, the conceptual error was in constructing logical arguments (C2), with a percentage of 11% (1 student). The procedural error was the solution not leading back to the statement being proved (P2), also 11% (1 student). Technical errors included mistakes in copying or simplifying expressions (T2), with a percentage of 22% (two students).

On Problem 3, the conceptual error was in using properties or theorems (C1), with a percentage of 11% (1 student). Procedural errors included mistakes in performing steps using proof methods or approaches (P1), with a percentage of 11% (1 student), and solutions not leading back to the statement being proved (P2), with a percentage of 11% (1 student). While technical errors involved mistakes in copying or simplifying expressions (T2), with a percentage of 11% (1 student).

Conceptual errors are mistakes in interpreting terms, defining concepts, or using incorrect/inappropriate formulas or properties [21]. There are two types of conceptual errors in mathematical proofs on real numbers, namely: errors in using properties or theorems, and errors in constructing logical arguments. The following are the conceptual errors made by the subjects.

1. Errors in using definitions, properties, or theorems (C1) are errors in choosing irrelevant theorems or properties, errors in applying theorems to conditions that do not meet the requirements, misunderstanding of formal definitions and theorems, and using the correct property at the wrong step [22]. Examples of errors in using definitions, properties or theorems are as follows. Example of errors in using properties or theorems is shown in Figure 2.

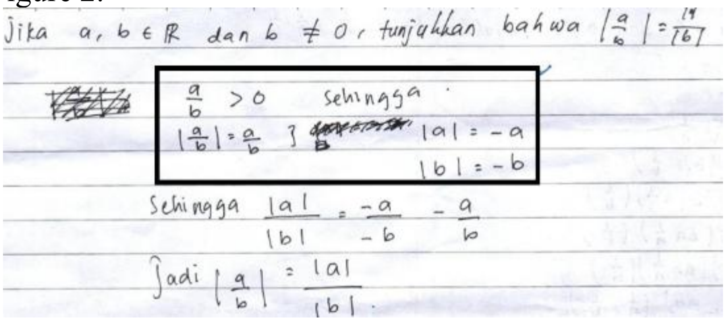

Figure 2. Error in Using Definitios

The student's mistake above is due to using an incorrect definition. The subject should have first defined $\left|\frac{a}{b}\right|$ using the correct absolute definition and then proceeded to prove $\frac{|a|}{|b|}$. However, the subject used the word "so that" and wrote the absolute signs $|a|$ and $|b|$, then interpreted them using the absolute definition with the assumption that a and b are negative. The argument is not general because it does not consider all conditions of a with $b \neq 0$. In addition, the statement $\left|\frac{a}{b}\right| = a/b$ is written without any justification that $a/b \geq 0$, so there is a logical gap. The results of the interview with the subject are as follows:

P: "You wrote this answer, right?"

S: "That's right, ma'am."

P: "Do you still remember why you wrote $\left|\frac{a}{b}\right| = a/b$?"

S: "Yes, ma'am, because I only proved the negative ones, ma'am, because if it's positive, the result is positive."

From the interview results, it can be seen that the definition of absolute value can be used directly without going through a formal definition. This shows that the subject does not yet understand deductive reasoning. The mistake made by the subject was due to a lack of understanding of general reasoning using formal definitions, in line with previous research which states that conceptual errors are caused by a lack of understanding of basic concepts [23].

2. Errors in constructing arguments are mistakes in establishing the relationship between premises and conclusions, resulting in a line of reasoning that is not logically valid and produces arguments that are inaccurate or invalid according to the rules of formal logic [24]. Example of errors in arranging arguments is shown in Figure 3.

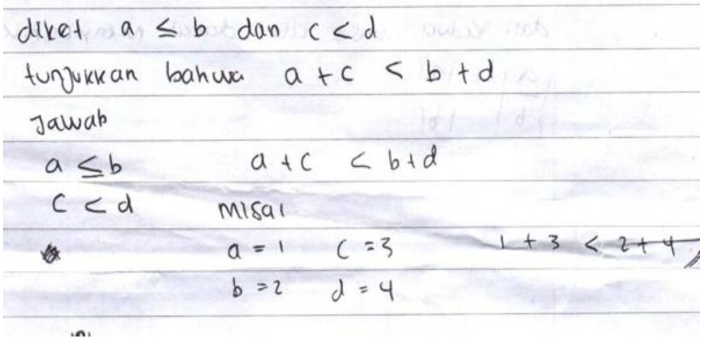

Figure 3. Error in Argument Construction

The student's mistake above is proving the statement by proving the statement through presenting examples. The subject takes numerical examples by substituting certain values into the variables that cause the above inequality to be true. The results of the interview with the subject are as follows.

P: "How did you show proof for this question?"

S: "I entered the numbers into variables a, b, c, and d."

P: "Was one example enough? Did you also try different numbers?"

S: "I only tried the numbers in the answer, and it was correct, so I didn't try other numbers."

The interview above shows that the subject interprets proof as being done by providing numerical examples. The subject also believes that one case can represent the truth for other cases. This is an indication that the subject does not yet understand deductive proof and its application to values that meet the conditions listed in the question. In this case, the subject was wrong in choosing the form of argument, wrong in reasoning the logical structure of the proof, and wrong in concluding that the example was proof. Thus, proving by providing a generalized example into a universal statement without proper deductive proof only shows possibility, but not universal proof [25].

Procedural errors are mistakes that occur due to the inability to carry out proof steps in a sequential and rule-based manner, even though the concepts and methods used are correct [26]. There are two types of procedural errors identified in this study, i.e. incorrectly performing steps when using proof methods or approaches, and solutions that do not lead back to the conclusion to be proved. The following are the procedural errors made by the subjects.

1. Errors in applying methods or approaches to proof constitute procedural errors, namely inaccuracies in executing the necessary steps in an approach to proof, even though the chosen method is correct[26]. Example of errors in performing steps when using proof methods or approaches is shown in Figure 4.

Figure 4. Error in Determining the Method

The subject's error is in the seventh line, where the subject writes $a + c < b + d$ without explaining where the answer comes from. However, the previous answer is in accordance with the inequality theorem that has been proven previously. The following is the result of the interview with the subject.

P: "Why did you write $a \leq b$ as $a - b \leq 0$ and $c < d$ as $c - d < 0$?"

S: "Because yesterday it was proven, ma'am, in the way I did it."

P: "Then in this line, why did you write $a + c < b + d$? Where did this come from?"

S: "For this one, because I thought that a-b is less than or equal to 0, which means it is not positive, so a is smaller than b. Similarly, c-d is less than zero, which means c is smaller than d, so I thought that if the smaller ones, a and c, are added together, the result will definitely be smaller than b and d, which are larger, so I wrote it like that."

P: "Then why is it written $a - b < d - c$ below?"

S: "Well, that's from $a + c < b + d$, then I moved the terms. And that matches lines 4, 5, and 6, right?"

T: "That's right, but that means it's not in order when you write it down?"

S: "I'm confused about how to order it, but I understand the meaning of the question."

The interview results above show that the subject understands the concept of inequalities and can perform addition on inequalities. However, the subject made a mistake in arranging the steps in a less systematic manner. The mistake in arranging the steps of the proof in a non-systematic manner due to the subject's failure to write down the steps in sequence is not because of a lack of understanding of the basic concepts, but rather because they are not accustomed to writing down the sequence formally [8].

2. The solution does not reach the conclusion being proved. Example of errors where solution fails to lead back to the proof is shown in Figure 5.

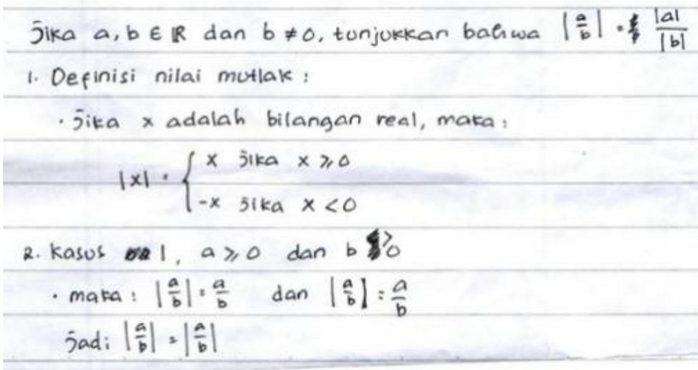

Figure 5. Error in Not Returning to The Proof

The subject's answer above shows that to answer question no. 3, the subject divided it into four cases. In the first case, the student made a mistake in that the answer did not lead to a conclusive proof. The student correctly wrote down the conditions $a \geq 0$ and $b > 0$ so that $\left|\frac{a}{b}\right| = \frac{a}{b}$ and wrote it down twice. However, at the end of the answer, the subject did not take the proof step towards $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ but wrote $\left|\frac{a}{b}\right| = \left|\frac{a}{b}\right|$. The interview with the subject is as follows.

P: "In the first case, you wrote $a \geq 0$ and $b > 0$, $\left|\frac{a}{b}\right| = \frac{a}{b}$ and $\left|\frac{a}{b}\right| = \frac{a}{b}$. Why did you write it this way?"

S: "Because a and b are positive, so the quotient is also positive, so $\left|\frac{a}{b}\right| = \frac{a}{b}$."

P: "Do you know that what is being proven is $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$?"

S: "Yes, ma'am. I focused on solving the left side because I think it is certain that $|a| = a$ and $|b| = b$."

The interview results show that the subject understands the definition of absolute value, understands case division, and knows that $|a| = a$ and $|b| = b$. However, the subject did not write down the proof steps completely and systematically. The subject neglected to organize the argument systematically, so the answer did not return to the conclusion [5].

Technical errors are mistakes that occur at the level of mechanically applying a step or manipulating symbols, such as miscalculations, incorrect signs, or misplacement of symbols, even though the concepts and procedures used are correct [27]. There are two types of technical errors in this study, i.e. errors in writing mathematical notation or symbols, and errors in copying or simplifying expressions. The following are the technical errors made by the subjects.

1. Errors in writing mathematical notation or symbols are mistakes in the writing or use of formal symbols in mathematical expressions or statements, which, even though the underlying concept is correct, still producing an incorrect symbolic form [28]. Example of errors in writing mathematical notation or symbols are shown in Figure 6.

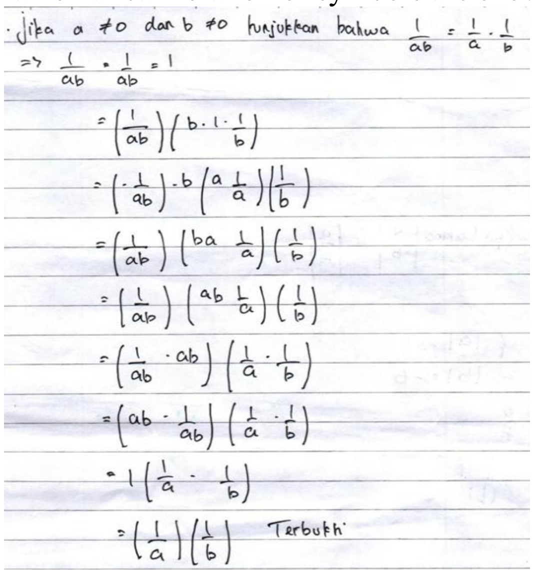

Figure 6. Error in Writing Mathematical Symbol

The error in the subject's answer in Figure 6 is in the second line, where the subject wrote $\frac{1}{ab} = \frac{1}{ab} = 1$ when it should have been written as $\frac{1}{ab} = \frac{1}{ab} \cdot 1$. However, the subject's answer up to the conclusion is correct. The following is the result of the interview with the subject.

P: "In this section, you wrote $\frac{1}{ab} = \frac{1}{ab} = 1$. What does $\frac{1}{ab} = 1$ mean?"

S: "Oh, it means $\frac{1}{ab} = \frac{1}{ab} \cdot 1$ because when you multiply a value by 1, the result is itself."

P: "Why does it have to be multiplied by 1?"

S: "I want to use the inverse property for the proof."

P: "Then why did you write it as an equality rather than a multiplication?"

S: "I wasn't careful enough, ma'am."

The interview results show that the subject understands that 1 is the identity of multiplication, the subject did not intend to state $\frac{1}{ab} = 1$, the error was not writing the multiplication symbol completely due to carelessness, and there were no misconceptions about the concept of algebra in the answer. Errors in writing symbols and notation that are not due to misconceptions but rather to carelessness are technical errors [29].

2. Errors in copying or simplifying expressions are mechanical mistakes that occur when students inaccurately transfer, copy, or simplify algebraic forms or numerical expressions, causing a change in the symbolic form that is not equivalent to the initial [30]. Example of errors in copying or simplifying expressions is shown in Figure 7.

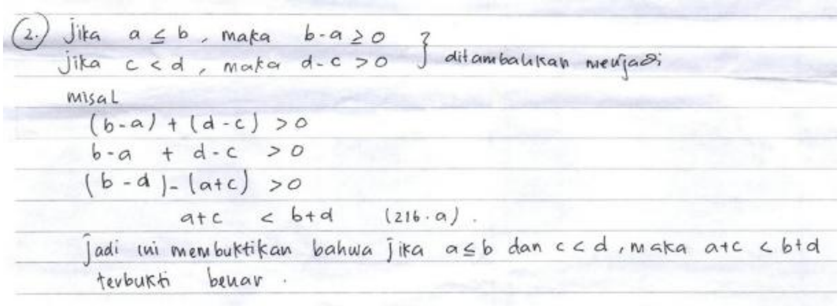

Figure 7. Error in Copying Algebraic Form

The subject's error is in the sixth line, which is an error in copying the algebraic form located in the sixth line, where it should be $(b + d) - (a + c) > 0$. The following is the result of the interview with the subject.

P: "In the step you wrote, $b - a + d - c > 0$ then changed to $(b - d) + (a + c) > 0$. Where did you get that form?"

S: "I saw the purpose of the proof was $a + c < b + d$, so I tried to group the terms together, combining b and d, then a and c."

P: "Do you think $b - a + d - c$ is the same as $(b - d) + (a + c)$?"

S: "Yes, I think it's the same, just a different grouping. But it seems I was wrong in giving a negative sign to d. I was not careful enough in grouping."

The interview results show that the subject understood the concept of inequalities, knew how to add inequalities, and understood the purpose of the proof. The subject made a mistake in manipulating the signs when grouping the terms due to carelessness. Technical errors included mistakes in writing symbols and algebraic operations due to a lack of thoroughness [31].

4. Conclusion

The results of the study show that in mathematical proofs on real numbers, students still make conceptual, procedural, and technical errors. The conceptual errors found are errors in using definitions and errors in using arguments caused by a lack of understanding of deductive proofs. The procedural errors found were errors in writing systematic steps and errors caused by answers that did not return to the conclusion due to difficulties in organizing arguments systematically. Meanwhile, the technical errors found were errors in writing notation and errors in copying simple expressions due to carelessness. The most dominant errors made by students were technical errors, particularly in writing mathematical notation or symbols. Further research or learning should focus on developing and testing effective learning strategies or models to minimize conceptual, procedural, and technical errors.

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