



## Analysis of The Critical Thinking Skills of Mathematics Education Students in Solving Differential Calculus Problems

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### ABSTRACT

*This study uses a descriptive qualitative approach with the aim of analyzing the critical thinking skills of 15 first-semester Mathematics Education students at Muhammadiyah University Sorong in solving problems in differential calculus material. The focus of this study stems from initial findings that indicate students' difficulties in understanding the concept of derivatives in story problems, which are thought to be caused by procedural errors and a lack of reflective skills in the thinking process. Specifically, this study aims to describe the profile of students' critical thinking skills based on Polya's indicators. The research used a descriptive qualitative approach with a written test consisting of three-story problems on December 17, 2025, and semi-structured interviews with six subjects (two high, two medium, two low), supported by a 1-4 scale assessment and Miles and Huberman's data analysis model with triangulation. The results reveal variations in ability: high-ability students consistently meet all four indicators; medium-ability students meet some indicators but are hampered by procedural problems; low-ability students have fundamental difficulties starting from understanding the problem. These findings emphasize the importance of differential calculus learning based on conceptual understanding, reflection, and contextual questions in Southwest Papua to improve students' critical thinking skills.*

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### 1. Introduction

Critical thinking skills are among the essential competencies in mathematics education because they are directly related to an individual's ability to assess, analyze, and make decisions rationally. In learning, critical thinking is understood not only as the ability to solve problems but also as a reflective reasoning process that involves consideration of relevant facts, concepts, methods, and criteria. Oktariani and Ekadiansyah (2020) emphasize that critical thinking enables students to consciously construct reasons for addressing



problems, which improves the quality of learning and prepares individuals for future academic and professional challenges [1].

In line with the demands of 21st-century learning, critical thinking has become a main competency required of prospective mathematics teachers. Differential calculus, especially the topic of derivatives, is one of the courses that demands this skill. The topic emphasizes mathematical precision and also requires students to relate the abstract concept of the derivative as a rate of change to real-world phenomena. Mastery of derivatives is an important foundation for applying mathematics in various disciplines, yet it remains a significant challenge for first-semester students in mathematics education programs in Indonesia.

Previous studies on derivatives can be grouped into three strands. The first strand concerns conceptual and procedural difficulties. Hakiki et al. (2025) found that students struggled with derivative word problems because of basic conceptual and procedural errors [2], and Mataheru et al. (2021) showed that procedural errors still dominate among mathematics education students [3]. Azzahra et al. (2025) likewise revealed procedural misconceptions about derivatives [8], and Supriyatin and Syafa'tun (2025) traced such difficulties to a weak understanding of limits [4]. Rahmawati et al. (2021) added that errors in solving problems and inaccurate results indicate insufficient mastery of basic concepts [10]. Taken together, these studies suggest that students' errors are not merely slips in computation but may reflect a lack of conceptual understanding. However, they classify errors after the fact and do not examine the reasoning process behind them.

The second strand concerns critical thinking in calculus. Siahaan and Sianipar (2020) showed that problem-based learning improves critical thinking in differential calculus [9], and Zetriuslita and Ariawan (2021) reported similar effects in integral calculus [7]. Adawiyah et al. (2021) examined critical thinking in solving differential equations by gender [6]. These studies mainly evaluate the effectiveness of instructional interventions or compare groups. They do not describe how students' critical thinking appears at each stage of solving a derivative problem.

The third strand uses Polya's problem-solving stages as an analytical lens. Sulistyorini and Napfiah (2019) showed that high-ability students could fulfill all four Polya indicators in integral calculus using the Riemann sum approach [5]. This strand shows that Polya's stages can reveal the structure of students' thinking, but it has not been connected to derivative word problems or to critical thinking explicitly. The three strands remain largely separate. Error studies identify what students get wrong, intervention studies show what improves critical thinking, and Polya-based studies show how students move through a solution, but few studies integrate them to describe critical thinking in derivative word problems. This is the gap addressed in the present study. To our knowledge, a stage-by-stage profile of first-semester students' critical thinking in derivative word problems, which would explain why procedural errors arise, is still lacking, particularly in the context of Southwest Papua.

Initial observations at Muhammadiyah University Sorong reinforce this gap. Some first-semester students fail to differentiate the function first and instead substitute variable values directly, even though most have mastered the basic differentiation procedure. As a result, the meaning of the rate of change asked for in the story problem cannot be interpreted correctly. This indicates a gap between technical procedural mastery and contextual critical thinking, a gap that error classification alone cannot explain.

Polya's four-stage framework [31] is appropriate for examining this gap because each stage requires a specific critical thinking skill, as described in Facione's core skills of interpretation, analysis, evaluation, inference, explanation, and self-regulation [32]. In this study, the indicators are operationalized as follows: 1) **Understanding the problem**

corresponds to *interpretation*. Students identify the known and asked quantities and interpret the meaning of “rate of change” in the context of the problem; 2) **Devising a plan** corresponds to *analysis and inference*. Students decide that the function must be differentiated first, choose the relevant derivative rules, and justify why the strategy fits the problem; 3) **Carrying out the plan** corresponds to *evaluation of procedure*. Students differentiate correctly, substitute values in the proper order, and check each step for accuracy, and 4) **Looking back** corresponds to *self-regulation and explanation*. Students verify the result, interpret it in the context of the problem, including units and reasonableness, and explain their reasoning.

This operationalization makes it possible to locate the stage at which critical thinking breaks down, for example when a student executes the procedure correctly at stage 3 but skipped the interpretation at stage 1.

Based on this description, the present study aims to describe the critical thinking skills of first-semester students in the Mathematics Education Study Program at Muhammadiyah University Sorong in solving differential calculus problems, using Polya indicators. Using a descriptive qualitative approach, with a written test of three story problems and triangulation interviews with six subjects selected by ability level, the study is expected to produce a comprehensive profile of students’ abilities. This profile can serve as a basis for recommendations for contextual differential calculus learning suited to the characteristics of the Southwest Papua region.

## Methods

This study employed a descriptive qualitative design to examine the critical-thinking performance of 15 first-semester students in the Mathematics Education Study Program at Universitas Muhammadiyah Sorong. All students in the single-class cohort, aged approximately 18–19 years, participated in a written test administered on December 17, 2025. The test consisted of three researcher-developed word problems on differential calculus. The problems and assessment rubric were reviewed by two lecturers and revised based on their suggestions before administration. The rubric assessed students' problem-solving processes through four Polya stages: understanding the problem, devising a plan, carrying out the plan, and checking the solution. Table 1 presents the corresponding critical-thinking indicators and scoring criteria, with scores ranging from 1 to 4 for each stage.

**Table 1. Guidelines for Assessing Students' Critical Thinking Skills in Solving Differential Calculus Problems**

| No | Polya Stage               | Problem Solving Indicators (Differential Calculus)                                | Critical Thinking Indicators                                | Assessment Indicators                                                                                                  | Score |
|----|---------------------------|-----------------------------------------------------------------------------------|-------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------|-------|
| 1  | Understanding the Problem | Identify what is requested and all important information from the given function. | Interpreting derivatives in the context of rates of change. | Does not recognize that the question is about derivatives/rates of change and important information is not identified. | 1     |
|    |                           |                                                                                   |                                                             | Recognizes that the question is related to derivatives, but the information gathered                                   | 2     |

|   |                       |                                                                                                      |                                                                |                                                                                                                                                  |   |
|---|-----------------------|------------------------------------------------------------------------------------------------------|----------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|---|
|   |                       |                                                                                                      |                                                                | is incomplete or incorrect.                                                                                                                      |   |
|   |                       |                                                                                                      |                                                                | Correctly identifies what is being asked and the main information, but does not explain it clearly.                                              | 3 |
|   |                       |                                                                                                      |                                                                | Completely identifies what is being asked and all important information, and explains the meaning of derivatives in the context of the question. | 4 |
| 2 | Developing plan       | a Determine the derivative concept/rules and the relevant sequence of steps for solving the problem. | Analyzing the interrelationship of concepts for a logical plan | There is no suitable plan, and the steps/formula are irrelevant.                                                                                 | 1 |
|   |                       |                                                                                                      |                                                                | Select several relevant rules, but the relationship between the steps and the objective is not yet clear.                                        | 2 |
|   |                       |                                                                                                      |                                                                | Determine the appropriate derivative rules and logical main steps even though the details are not yet complete.                                  | 3 |
|   |                       |                                                                                                      |                                                                | Develop a coherent and comprehensive plan, select the most appropriate derivative rules, and provide a brief justification for each step.        | 4 |
| 3 | Implementing the plan | Perform derivative procedures and algebraic manipulations according to plan.                         | Assessing the accuracy of the calculation process.             | Many conceptual/procedural errors were made, rendering the results unreliable.                                                                   | 1 |
|   |                       |                                                                                                      |                                                                | The steps were relevant, but there were several conceptual/procedural errors that affected the results.                                          | 2 |

|   |                         |                                                                          |                                                                |                                                                                                                                                   |   |
|---|-------------------------|--------------------------------------------------------------------------|----------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------|---|
|   |                         |                                                                          |                                                                | The calculations were logical and generally correct, with only minor errors that did not alter the main method.                                   | 3 |
|   |                         |                                                                          |                                                                | The calculations were systematic and correct, the notation was clear, and minor errors appeared to have been checked and corrected.               | 4 |
| 4 | Checking and concluding | Double-check the results and state conclusions according to the context. | Draw conclusions and assess the reasonableness of the results. | No clear conclusion or conclusion does not match the results/context.                                                                             | 1 |
|   |                         |                                                                          |                                                                | Only writes numerical/symbolic answers without contextual meaning and without checking for plausibility.                                          | 2 |
|   |                         |                                                                          |                                                                | Composes conclusions that are accurate and contextually appropriate, even though plausibility checks are not clearly stated.                      | 3 |
|   |                         |                                                                          |                                                                | Composes complete conclusions in their own words and relates them to the context, as well as explicitly checking the plausibility of the results. | 4 |

The scores for the four Polya stages were summed for each question and then combined across all three questions to obtain a cumulative score ranging from 12 to 48. Students were classified as having high (36–48), moderate (24–35), or low (12–23) critical-thinking ability. Two students from each category were purposively selected for semi-structured interviews, resulting in six participants (ST1, ST2, SS1, SS2, SR1, and SR2). Interview recordings were transcribed and analyzed alongside the corresponding written responses to examine students' reasoning, selection and application of differentiation rules, and approaches to checking their answers. Scores were used descriptively to contextualize students' performance, while interpretations of their critical-thinking processes were based primarily on evidence from written responses and interviews.

## 2. Result and Discussion

This section presents the results of an analysis of the critical thinking abilities of six selected students in the Differential Calculus course based on a written test consisting of three-story questions and in-depth interviews. The six subjects were coded ST1 and ST2 in the high ability category, SS1 and SS2 in the moderate ability category, and SR1 and SR2 in the low ability category. Data analysis followed a qualitative descriptive approach with triangulation of sources between written test results and interview transcripts, which were presented in an integrated manner per question to identify patterns of critical thinking ability between categories.

### Question 1

In question 1 regarding the speed of a car on a straight road, the ability profiles of the six subjects based on Polya's indicators are presented in Table 2, while the subjects' original answers are shown in Figures 1-6.

**Table 2. Critical Thinking Ability Profiles of 6 Subjects on Question 1**  
(T1 = Understanding the Problem, T2 = Formulating a Plan, T3 = Implementing the Plan, T4 = Checking & Drawing Conclusions)

| Subject | T1 | T2 | T3 | T4 | Total | Description |
|---------|----|----|----|----|-------|-------------|
| ST1     | 4  | 4  | 4  | 4  | 16    | High        |
| ST2     | 4  | 4  | 4  | 4  | 16    |             |
| SS1     | 4  | 4  | 2  | 2  | 12    | Medium      |
| SS2     | 4  | 4  | 4  | 4  | 16    |             |
| SR1     | 4  | 3  | 2  | 2  | 11    | Low         |
| SR2     | 3  | 2  | 2  | 1  | 8     |             |

$$1.) y = \frac{5}{(4+2x)^3} = 5 \cdot (4+2x)^{-3} \quad \text{jika dik, } x=2$$

$$y' = -15(2)(4+2x)^{-4}$$

$$y' = \frac{-30}{(4+2x)^4}$$

$$y' = \frac{-30}{(4+2 \cdot 2)^4} = \frac{-30}{(4+4)^4} = \frac{-30}{8^4}$$

Figure 1. Subject ST1 working on question number 1

$$1.) y = \frac{5}{(4+2x)^3} \quad \text{jika } x=2$$

$$f(x) = \frac{5}{(4+2x)^3} = 5(4+2x)^{-3}$$

$$f'(x) = 5(-3)(4+2x)^{-4}$$

$$f'(2) = 5(-3)(4+2 \cdot 2)^{-4}$$

$$f'(2) = -30(4+2 \cdot 2)^{-4}$$

$$f'(2) = -30(8)^{-4} = \frac{-30}{8^4} = \frac{-30}{4096}$$

Figure 2. Subject ST2 working on question number 1

$$1.) y = \frac{5}{(4+2x)^3}$$

$$y' = 5 \cdot (4+2x)^{-3}$$

$$= -15 \cdot 2 \cdot (4+2x)^{-4}$$

$$= -30 + (4+2x)^{-4}$$

$$= -30 + (8)^{-4} = \frac{-30}{8^4}$$

Figure 3. Subject SS1 working on question number 1

$$1.) y = \frac{5}{(4+2x)^3}$$

$$y' = \frac{5}{(4+2x)^3} \cdot 5(4+2x)^{-3}$$

$$= -15(2)(4+2x)^{-4}$$

$$= -30(4+2x)^{-4}$$

$$= \frac{-30}{(4+2x)^4} = \frac{-30}{(4+2 \cdot 2)^4} = \frac{-30}{8^4}$$

Figure 4. Subject SS2 working on question number 1

$$1.) y = \frac{5}{(4+2x)^3}$$

$$y' = \frac{5}{(6x^2) = 6x \cdot 6x \cdot 6x}$$

$$y' = \frac{5}{36}$$

$$y' = \frac{5}{36} \cdot x^2$$

Figure 5. Subject SR1 working on question number 1

$$1.) y = \frac{5}{(4+2x)^3}$$

$$u = (4+2x)$$

$$u' = 4+6x$$

$$f = 15$$

$$\frac{f u'}{u^4} = \frac{10(4+6x)}{(4+2x)(4+6x)^2} = \frac{11+60x}{(4+2x)(4+6x)^2}$$

Figure 6. Subject SR2 working on question number 1

In Question 1, which concerned the speed of a car traveling on a straight road, ST1, ST2, and SS2 achieved the maximum score of 16 on all four stages of Polya's problem-solving model (Table 2; Figures 1, 2, and 4). Their written answers indicated that they possessed optimal skills in identifying relevant information, selecting the appropriate differentiation rules, performing calculations, and drawing conclusions. However, interview findings revealed differences in their understanding. ST1 could explain the procedures for differentiation and substitution but remained uncertain about the correctness of their answer even after solving the problem correctly. ST2 could describe the solution procedure but tended to rely on formulas and methods demonstrated by the instructor. In contrast, SS2 was able to explain the relationship between derivatives and the rate of change with respect to time, justify the chosen differentiation rule, and describe how the result was verified within the context of the problem. These differences indicate that identical written scores do not always reflect the same level of confidence or the same ability to explain and evaluate mathematical solutions. Riyanto and Ishartono (2022) associate high critical thinking ability with the integration of conceptual understanding and the evaluation of solution outcomes [12]. Similarly, Dewanti (2018) explains that "trial-and-error" strategies can help students identify correct guesses and evaluate solutions reflectively [13].

Unlike students in the high-performing category, SS1 showed a decline in performance during the stages of implementing the plan and checking and concluding the results (Table 2; Figure 3). During the implementation stage, SS1 applied the relevant differentiation rules but made calculation errors, resulting in an inaccurate answer. During the checking and conclusion stage, SS1 presented the final answer without explaining its meaning in the context of the problem or verifying its validity. Further interview findings revealed that SS1 had difficulty explaining the rationale behind the choice of differentiation strategies and tended to rely on methods previously demonstrated by the instructor. These findings indicate that applying the correct differentiation rules does not always guarantee accurate calculations or effective evaluation of solutions. This discrepancy highlights the importance of evaluating mathematical procedures and drawing logical conclusions in critical thinking [14].

More fundamental difficulties were observed in SR1 and SR2 (Figures 5 and 6). Their written responses revealed difficulties in selecting appropriate differentiation rules and organizing coherent solution steps. The interviews further indicated confusion about derivative notation and the relationships between successive steps. Both students tended to wait for guidance and struggled to explain the reasoning behind their chosen procedures. These difficulties suggest that limited conceptual understanding of derivatives, combined with difficulties interpreting word problems, hindered their ability to formulate and carry out appropriate solution strategies. Unlike SS1, whose difficulties were more evident during the implementation and verification stages, SR1 and SR2 encountered problems from the initial stages of understanding the problem and planning a solution. These findings are consistent with Lea et al. (2022), who reported that students with lower ability in differentiation often struggle to represent derivative concepts in different mathematical forms and apply them appropriately to problem contexts [15]. Ismayanti et al. (2022) also noted that low self-confidence in mathematics learning may lead students to give up easily, feel incapable of solving problems, and depend on others, ultimately affecting the quality of their problem-solving processes [16]. In the present study, the most evident difficulties among SR1 and SR2 concerned conceptual understanding and mathematical reasoning.

## Question 2

For question 2 regarding the rate of change in wave height, the critical thinking ability profiles of the six subjects based on Polya's indicators are presented in Table 3, while the original answers of the six subjects are shown in Figures 7-12.

**Table 3. Critical Thinking Ability Profiles of 6 Subjects in Question 2**

| Subject | T1 | T2 | T3 | T4 | Total | Description |
|---------|----|----|----|----|-------|-------------|
| ST1     | 4  | 4  | 4  | 4  | 16    | High        |
| ST2     | 4  | 4  | 4  | 4  | 16    |             |
| SS1     | 4  | 4  | 4  | 4  | 16    | Medium      |
| SS2     | 4  | 4  | 2  | 2  | 12    |             |
| SR1     | 4  | 4  | 2  | 2  | 12    | Low         |
| SR2     | 4  | 2  | 2  | 2  | 10    |             |

**Figure 7. Subject ST1 working on question number 2**

**Figure 8. Subject ST2 working on question number 2**

**Figure 9. Subject SS1 working on question number 2**

**Figure 10. Subject SS2 working on question number 2**

**Figure 11. Subject SR1 working on question number 2**

**Figure 12. Subject SR2 working on question number 2**

Based on Table 3, there are differences in problem-solving abilities among subjects at each stage of Polya's model. In question number 2, subjects ST1, ST2, and SS1 obtained maximum scores in all stages of problem solving according to Polya, as shown in Figures 7, 8, and 9. This indicates that the three subjects successfully understood the problem through identifying the context of the rate of change of wave height and relevant information from the given function model. Furthermore, the three subjects were also able to plan the solution by determining the appropriate strategy and derivative rules and implement the plan through the process of trigonometric function reduction and determining the rate of change at the requested time accurately. In addition, the three subjects rechecked the solution results by relating the calculation results to the problem context.

The interview results show that subjects ST1, ST2, and SS1 are able to explain the meaning of the rate of change of wave height as a derivative function over time, state the reasons for choosing the derivative rule used and explain the steps for checking answers by relating the calculation results back to the context of the problem. This pattern of solution shows consistency in the application of all stages of Polya and reflects good critical thinking skills. These findings are in line with Rejeki et al. (2021), who state that individuals with high critical thinking skills tend to consciously reflect on and evaluate the solution strategies used, rather than simply following mechanical procedures [17]. Furthermore, these findings are reinforced by Mukarrohmah et al (2024), who emphasize that high problem-solving ability in derivative material is characterized by a strong understanding of concepts and regularity in each stage of the solution [18].

Subjects SS2 and SR1 scored lower than the previous group, particularly in stages of implementing the plan and reviewing the results. As shown in Figure 10 and Figure 11, both subjects made mistakes in deriving trigonometric functions involving root forms and lacked precision in evaluating their work, resulting in answers that did not meet the requirements of the questions. The interview results indicate that although both subjects were able to understand the problem and recognize the relationship between the question and the concept of derivatives, they experienced confusion in implementing the solution plan, difficulty in explaining the steps of the function derivation, and tended not to re-verify the calculation results. These findings are consistent with the research by Mirza et al. (2022), which states that some subjects were able to determine the initial steps of the solution but had difficulty explaining the logic behind the use of formulas, so that the solution process tended to be algorithmic [19]. This is reinforced by Andayani et al. (2024), who state that carelessness in calculation details is one of the main causes of inconsistency in students answers on derivatives and trigonometric material [20].

Meanwhile, subjects SR2 showed more fundamental difficulties. This subject was unable to understand the problem as a whole, particularly in relating the context of tides to the concept of rate of change, as shown in Figure 12. This is reflected in the low scores on several stages of problem solving, especially in the planning and implementation stage, so that the answers produced did not reflect a comprehensive understanding of the problem. The interview results reveal the uncertainty of subject SR2 in understanding the essence of derivatives and the relationship between the function model and the tidal phenomenon, accompanied by low self-confidence as seen from the doubt about the steps chosen and the uncertainty of the final result. This finding is in line with Hajerina et al. (2022), who stated that a weak understanding of derivative concepts cause students to have difficulty determining the appropriate solution strategy, where weak conceptual understanding is a common characteristic of subjects with low critical thinking skills [21].

### Question 3

For question number 3 regarding the rate of change in antenna slope, the critical thinking ability profiles of the six subjects based on Polya's indicators are presented in Table 4, while the subjects original answers are shown in Figure 13-18.

**Table 4. Critical Thinking Ability Profiles of 6 Subjects in Question 3**

| Subject | T1 | T2 | T3 | T4 | Total | Description |
|---------|----|----|----|----|-------|-------------|
| ST1     | 4  | 4  | 4  | 4  | 16    | High        |
| ST2     | 4  | 4  | 4  | 4  | 16    |             |
| SS1     | 3  | 2  | 2  | 2  | 9     | Medium      |
| SS2     | 3  | 2  | 1  | 2  | 8     |             |
| SR1     | 3  | 2  | 1  | 1  | 7     | Low         |
| SR2     | 3  | 1  | 1  | 1  | 6     |             |

$$\begin{aligned}
 3) \quad y &= 3 \arctan(2x) + 2 \arcsin(x) & \text{jika dir: } x &= \frac{1}{2} \\
 u &= 2 & u' &= 1 \\
 y' &= 3 \cdot \frac{1}{1+u^2} \cdot \frac{du}{dx} + 2 \cdot \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx} \\
 y' &= \frac{3}{1+(2x)^2} \cdot 2 + \frac{2}{\sqrt{1-(x)^2}} \cdot 1 \\
 y' &= \frac{6}{1+4x^2} + \frac{2}{\sqrt{1-x^2}} \\
 y' &= \frac{6}{1+4(\frac{1}{2})^2} + \frac{2}{\sqrt{1-(\frac{1}{2})^2}} = \frac{6}{1+4 \cdot \frac{1}{4}} + \frac{2}{\sqrt{1-\frac{1}{4}}} = \frac{6}{1+1} + \frac{2}{\sqrt{\frac{3}{4}}} = \frac{6}{2} + \frac{2}{\frac{\sqrt{3}}{2}} = 3 + \frac{4}{\sqrt{3}}
 \end{aligned}$$

Figure 13. Subject ST1 working on question number 3

$$\begin{aligned}
 3) \quad y(x) &= 3 \arctan(2x) + 2 \arcsin(x) & \text{dik: } x &= \frac{1}{2} \\
 y'(x) &= 3 \cdot \frac{1}{1+u^2} \cdot \frac{du}{dx} + 2 \cdot \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx} \\
 &= 3 \cdot \frac{1}{1+4} \cdot 2 + \frac{2}{\sqrt{1-u^2}} \cdot 1 \\
 &= \frac{6}{5} + \frac{2}{\sqrt{1-u^2}} \\
 \text{jika } \frac{6}{5} + \frac{2}{\sqrt{1-u^2}} &= \frac{6}{5} + \frac{2}{\sqrt{1-(\frac{1}{2})^2}} = \frac{6}{5} + \frac{2}{\sqrt{\frac{3}{4}}} = \frac{6}{5} + \frac{4}{\sqrt{3}}
 \end{aligned}$$

Figure 14. Subject ST2 working on question number 3

$$\begin{aligned}
 3) \quad y &= 3 \arctan(x) + 2 \arcsin(x) \\
 u &= 3 \arctan(x) & u' &= 3 \\
 u &= 2 \arcsin(x) & u' &= 2 \\
 \frac{du}{dx} &= 3 & \frac{du}{dx} &= 2 \\
 y' &= 3 \cdot \frac{1}{1+u^2} \cdot \frac{du}{dx} + 2 \cdot \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx} \\
 y' &= \frac{3}{1+x^2} \cdot 3 + \frac{2}{\sqrt{1-x^2}} \cdot 2 \\
 y' &= \frac{9}{1+x^2} + \frac{4}{\sqrt{1-x^2}}
 \end{aligned}$$

Figure 15. Subject SS1 working on question number 3

$$\begin{aligned}
 3) \quad y(x) &= 3 \arctan(x) + 2 \arcsin(x) \\
 y'(x) &= 3 \cdot \frac{1}{1+u^2} \cdot \frac{du}{dx} + 2 \cdot \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx} \\
 &= 3 \cdot \frac{1}{1+x^2} \cdot 1 + \frac{2}{\sqrt{1-u^2}} \cdot 1 \\
 &= \frac{3}{1+x^2} + \frac{2}{\sqrt{1-x^2}}
 \end{aligned}$$

Figure 16. Subject SS2 working on question number 3

$$\begin{aligned}
 3) \quad y &= 3 \tan(2x) + 2 \sin(x) \\
 y &= 3 \tan(2x) + 2 \\
 y &= 6x + 2 \\
 y &= 6x \\
 x &= \frac{1}{6}(6x+2) \\
 &= 2x + \frac{2}{6} \\
 &= 2x + \frac{1}{3} \\
 &= x + \frac{1}{3} \\
 &= 3
 \end{aligned}$$

Figure 17. Subject SR1 working on question number 3

$$\begin{aligned}
 3) \quad y &= 3 \arctan(2x) + 2 \arcsin(x) \\
 \frac{dy}{dx} &= \frac{u'}{1+u^2} \\
 x &= 3 \\
 u &= (2x) + 2 \arcsin(x) \\
 u &= (2x) + 2 \arcsin(x) \\
 \frac{dy}{dx} &= \frac{(2x) + 2}{(1+4x^2)(2x) + 2} = \frac{(2x) + 2}{(1+4x^2)(2x) + 2}
 \end{aligned}$$

Figure 18. Subject SR2 working on question number 3

In question number 3, subjects ST1 and St2 maintained optimal scores in all stages of Polya's problem-solving process, as shown in Figures 13 and Figure 14. In the problem-understanding stage, both subjects were able to interpret the rate of change in the antenna slope as the derivative of an inverse trigonometric function with respect to time and identify relevant information from the given function model. Furthermore, in the planning stage, ST1 and ST2 determined the correct derivative rules, then systematically implemented the plan through accurate parameter substitution. In the final stage, both subjects double-checked to ensure that the calculation results were appropriate for the context of the problem.

The interview results deepened the characterization of these individuals. Subjects ST1 described the solution procedure fluently and with absolute confidence, even stating her readiness to defend her solution against her colleague's alternative through validation of its conceptual foundation. In contrast, ST2 achieved similar accuracy but acknowledged caution and doubt in the final stage. This reflective pattern was consistent across previous questions, which indicated inherent critical thinking characteristics rather than weaknesses in conceptual understanding. This finding is in line with Ariyana et al. (2018), who stated that critical thinking requires reflection on reasoning and the relevance of solutions to the problem context [22]. Student with high-level critical thinking skills are usually able to adjust their strategies when inconsistencies arise and produce more meaningful solutions. This is supported by Sulistyaningsih et al. (2025), who noted that students with adequate prior knowledge tend to display structured and stable problem solving, particularly in differential calculus, where they not only find the correct answer but also maintain their arguments and are able to logically explain the process [23]. Fitriana et al. (2022) emphasize

that critical thinking skills not only increase intelligence but also facilitate effective problem solving [24].

In contrast, subjects SS1 and SS2 showed a decline in performance compared to questions 1 and 2. Although in the understanding and planning stages, both subjects were able to identify that the problem was related to function derivatives and select the relevant rules, crucial errors occurred in the execution stage. These errors occurred in the process of reducing inverse trigonometric functions, particularly when determining the derivative of the function  $2 \arcsin(x)$ . Both subjects incorrectly use the wrong form, namely  $\frac{1}{\sqrt{1+u^2}}$  instead of  $\frac{1}{\sqrt{1-u^2}}$ , as shown in Figure 15 and Figure 16. This error had a direct impact on the inaccuracy of the final result, which illustrates the imbalance between a strong understanding of basic concepts and an unstable mastery of inverse trigonometric formulas.

The interview results reinforce this pattern, in which SS1 and SS2 tend to apply memorized formulas directly without verifying their validity, and rarely re-evaluate their calculations. These findings are consistent with Nuraini et al. (2019), who stated that subjects with moderate mathematical abilities often neglect to re-check their final results, making the final results prone to errors [25]. This is reinforced by Xiong et al. (2025), who revealed that weaknesses in performance control and self-reflection contribute to the inaccuracy of solution strategies, especially when self-correction fails [26].

Meanwhile, subjects SR1 and SR2 faced more fundamental difficulties. At the problem comprehension stage, neither of them was able to fully relate the problem context to the derivative concept, while their mastery of inverse trigonometric derivative formulas was also not evident. Although the initial interviews showed that SR1 and SR2 generally understood the requirements of the question, on the written answer sheet, both immediately substituted the parameter values into the initial function without performing the function reduction process, as shown in Figure 17 and Figure 18. This direct approach strategy triggered a chain of systematic errors that lasted from the question interpretation stage to the final evaluation.

Further interviews revealed that SR1 and SR2 showed confidence when applying memorized formulas, but experienced significant difficulties when asked to perform further reasoning or explain the conceptual basis. This pattern is in line with Hasanah (2019), who identified that difficulties in derivative material often stem from an inability to analyze the correct derivation technique, limitations in mastering basic mathematical operations, and misunderstanding the essence of the problem at hand [27]. Lusiana (2017) adds that such errors not only reduce academic achievement but also reflect fundamental obstacles in the learning process [28]. In addition, Hari et al. (2018) report similar difficulties in critical thinking indicators such as argument checking and answer validity [29]. Meanwhile, Hidayah et al. (2025) emphasize that students with low critical thinking skills have not achieved optimal results and still require direct guidance from teachers [30].

Overall, the critical thinking ability profile of first-semester students in the Mathematics Education Study Program at Muhammadiyah University Sorong in solving differential calculus problems shows a clear range of abilities based on three contextual story problems, with the high category consistently excelling on all Polya indicators through deep conceptual understanding and systematic reflection, the medium category has a range of procedural errors even though the basic concepts have been understood, and the low category has limited superficial understanding without adequate problem-solving strategies. This variation underscores that success is not merely about memorizing derivative formulas, but rather the integration of problem interpretation, analysis of the interrelationship of rate of change concepts, and adaptive reflective evaluation of the complexity of inverse trigonometry and mixed functions. These findings provide a crucial foundation for the design

of differential learning in the Southwest Papua region, which should prioritize tiered scaffolding, authentic questions based on local phenomena, and self-correction exercises to bridge the procedural-conceptual gap to optimize the critical thinking of prospective mathematics teachers.

### 3. Conclusion

This study reveals a distinct variation in how first semester Mathematics Education students at Universitas Muhammadiyah Sorong apply critical thinking when solving differential calculus problems. High performing students consistently navigate all four stages of Polya's problem solving framework. Beyond executing differentiation and numerical calculations accurately, they demonstrate a solid grasp of contextual rates of change and actively evaluate their reasoning throughout the solution process.

In contrast, students in the moderate ability group generally grasp the core objective of each problem and understand fundamental derivative concept, yet they encounter procedural hurdles when working with more complex structures such as inverse trigonometric functions. Meanwhile, low ability students experience more deep seated difficulties. A weak conceptual foundation impedes their ability to select appropriate strategies, leading them to approach problems mechanically without fully making sense of the underlying mathematical steps.

These findings underscore that critical thinking in differential calculus goes well beyond memorizing formulas or executing algebraic steps. Rather, it hinges on a student's ability to interpret problem contexts, interrelate mathematical concepts and reflect critically on their final outputs.

To address these gaps, initial calculus instruction should place greater emphasis on conceptual understanding through authentic, real world problems that resonate with students' experiences. Instructors are encouraged to adopt adaptive scaffolding strategies tailored to varying student readiness levels, while actively fostering habits of self checking and reflection. Future research could further investigate the impact of specific instructional models such as problems based learning or contextual frameworks rooted in the local context of Southwest Papua to cultivate students' critical thinking skills more effectively.

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